18-Aug-2021

The Head of Department,
Department of Computer Science,
St. Joseph's College (Autonomous)
Trichy, India. 620 002.

Sub : Campus Recruitment Engagement Proposal

Dear Sir,

Our Company 20Cube Logistics Solutions Pvt. Ltd., was incorporated in 2011 and we are in to Supply Chain related technology developments.

Based on our discussions with you, we are happy to inform you that we wish to engage your esteemed institution for our campus recruitment engagement for next 3 years. We require your support to identify a suitable last year graduates from MCA and MSc Computer Science departments. The recruitment process location will be either at your college premise or our Chennai facility, which will be communicated to you 15 days in advance.

The Selected candidates will be offered by the HR Consulting firm **"TalentPro India HR Pvt Ltd.,"**

Thanking you,
For **20Cube Logistics Solutions Pvt. Ltd.,**

SALETH JOHNPAUL
Vice President – Information Technology

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Arclines
Visualise Beyond

12, September 2020

Head of the Department,
Department of Computer Science,
St Joseph's College (Autonomous),
Tiruchirappalli – 620 002.

Sub: Linkage Proposal – reg

Dear Sir,

Our Company ARCLINES Visualize Beyond, is situated at Tiruchirappalli and we are into indigenous AR and VR Application development and Implementation.

We are pleased to accept your request to be a training partner for your PG and UG Programmes. We herebyundertake to provide our expertise in the areas of Curriculum Design, Faculty Training, Internship and placement as part of this agreement for a periodof three years.

On behalf of ARCLINES Visualize Beyond, I sign this agreement dated 12th, September 2020

Thanking you,

For of ARCLINES Visualize Beyond,

R PRAKASH

Managing Partner

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01, August 2020

Head of the Department,
Department of Computer Science,
St Joseph's College (Autonomous),
Tiruchirappalli – 620 002.

Sub: Linkage Proposal – reg

Dear Sir,

Our Company Pyroferus Technologies., is situated at Chennai, incorporated in the year 2011. We have vast experience in working across multiple verticals like Banking, Finance, Insurance, and Retail, to name a few and across platforms.

We are pleased to accept your request to be a training partner for your PG Programme. We hereby undertake to provide our expertise in the areas of Curriculum Design, Faculty Training, Internship and placement as part of this agreement for a period of two years.

On behalf of Pyroferus Technologies, I sign this agreement dated 1st, August 2020

Thanking you,

For of Pyroferus Technologies,

Somasundar M
VP - Technology

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Advances in Mathematics: Scientific Journal **9** (2020), no.5, 2791–2800
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.5.40

FIXED POINTS THEOREMS OF (κ, μ) RATIONAL CONTRACTIVE MAPPINGS IN ORDERED COMPLEX VALUED QUASI METRIC SPACES

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ABSTRACT. In this article, we proved so many fixed point results with help of new notion (κ, μ) rational contractive mappings in ordered complex valued quasi metric spaces and show that the example exist as well as application on fixed point theorems.

1. INTRODUCTION

The Banach contraction principle is a basic tool for developing the fixed point results. Many authors contributed for proving fixed point results [1–5]. Doitchinov in [8], Adam et al. in [4], Dung in [10] have introduced fixed point theorems existence of complex valued quasi metric spaces. The concept of almost contraction initiated by Berinide. So many authors generalized that contraction, [6, 7].

Before entering into our main results we shall recall some basic definition and results which are needful.

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2010 *Mathematics Subject Classification.* 54E35.
Key words and phrases. partially ordered metric spaces, complex valued quasi metric spaces, fixed point.

2. Preliminaries

We recollect some basic definitions and notions which is useful for proving our main results.

Let $\mathbb{C}$ be the set of complex numbers and $p_1, p_2 \in \mathbb{C}$. Define a partial order $\leq$ on $\mathbb{C}$ as follows:

$p_1 \leq p_2$ if and only if $Re(p_1) \leq Re(p_2)$,$Im(p_1) \leq Im(p_2)$.
Consequently, one can infer that $p_1 \leq p_2$ if one of the following conditions is satisfied:

(i) $Re(p_1) = Re(p_2)$, $Im(p_1) < Im(p_2)$,
(ii) $Re(p_1) < Re(p_2)$, $Im(p_1) = Im(p_2)$,
(iii) $Re(p_1) < Re(p_2)$, $Im(p_1) < Im(p_2)$,
(iv) $Re(p_1) = Re(p_2)$, $Im(p_1) = Im(p_2)$.

In particular, we write $p_1 \lneqq p_2$ if $p_1 \neq p_2$ and one of $(i), (ii)$ and (iii) are satisfied and we write $p_1 < p_2$ if only (iii) is satisfied. Notice that

(a) If $0 \leq p_1 \lneqq p_2$, then $|p_1| < |p_2|$,
(b) If $p_1 \leq p_2$ and $p_2 < p_3$ then $p_1 < p_3$,
(c) If $a, b \in R$ and $a \leq b$ then $ap_1 \leq bp_1$ for all $p \in \mathbb{C}$.

Definition 2.1. *A complex quasi metric on a non-empty set X is a function $\psi_{cp} : X \times X \to \mathbb{C}$ such that for all $x, y, z \in X$:*

(1) $\psi_{cp}(x, y) = 0$ *if and only if* $x = y$,
(2) $\psi_{cp}(x, y) \leq \psi_{cp}(x, z) + \psi_{cp}(z, y)$.

Definition 2.2. *Let (X, ψ_{cp}) be a complex quasi metric space*

(1) *Let $\{x_n\}$ be a cauchy sequence if for every $0 \prec c \in \mathbb{C}$ find a integer N such that $\psi_{cp}(x_n, x_m) \prec c$ for every $m, n \succcurlyeq N$.*
(2) *Let $\{x_n\}$ converges to an element $x \in X$ if for every $0 \prec c \in \mathbb{C}$ find a integer N such that $\psi_{cp}(x_n, x) \prec c$ for all $n \succcurlyeq N$.*
(3) *Suppose that (X, ψ_{cp}) is complete if for every cauchy sequence in X converges to a point in X.*

Definition 2.3. *The function $\mu : [0, \infty) \to [0, \infty)$ is said to be an altering distance function if the following conditions are satisfied:*

(i) *μ is continuous and increasing;*
(ii) $\mu(a) = 0$ *iff* $a = 0$.

So many authors discussed alerting distance function. Khan et al. in [9] introduced the concept of altering distance function. Here we introduced new notion (κ, μ) rational contractive mappings in ordered complex valued quasi metric spaces where κ and μ are the altering distance function.

3. Main Results

In this section, we prove our main results.

Definition 3.1. *Let $(X, \leq, \psi_{cp})$be an ordered Quasi metric space. Let κ and μ be altering distance functions. Then the mapping $g : X \to X$ is an (κ, μ) rational contraction mapping if there exists $M \geq 0$ such that:*

$$\kappa(\psi_{cp}(gx, gy)) \leq \kappa(R(x, y)) - \mu(R(x, y)) + M\kappa(S(x, y)) \tag{3.1}$$

where $R(x, y) = max\{\psi_{cp}(x, y), \dfrac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1 + (\psi_{cp}(x, y))^2}, \dfrac{\psi_{cp}(x, gy)\psi_{cp}(y, gy)}{1 + \psi_{cp}(x, y) + \psi_{cp}(y, gy)}\}$
and $S(x, y) = min\{\dfrac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1 + \psi_{cp}(x, y)}, \dfrac{\psi_{cp}(x, gy)\psi_{cp}(y, gx)}{1 + \psi_{cp}(x, y)}\}$
for all comparable $x, y \in X$.

Theorem 3.1. *Let $(X, \leq, \psi_{cp})$ be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let $g : X \to X$ be a increasing continuous mapping with respect to $\leq$. Suppose that g is an (κ, μ)- rational contractive mapping for all comparable $x, y \in X$ then g has a fixed point.*

Proof. It should be shown that g has a fixed point. Let us consider $x_0$be a point in X. We define a sequence $\{x_l\}$ in X such that $x_{l+1} = gx_l$.
Since g is a increasing sequence, $x_0 \leq gx_0 = x_1 = gx_0 \leq x_2 = gx_1$.
Again $x_1 \leq x_2$ and g is a increasing therefore by induction we show that:
$x_0 \leq x_1 \leq ..... \leq x_l \leq x_{l+1} \leq ....$ Consider $x_l \neq x_{l+1}$ for every $l \in N$. So from the equation (3.1) we have:

$$\begin{aligned} \kappa(\psi_{cp}(x_l, x_{l+1})) &= \kappa(\psi_{cp}(gx_{l-1}, gx_l)) \leq \\ &\leq \kappa(R(x_{l-1}, x_l)) - \mu(R(x_{l-1}, x_l)) + M\kappa(S(x_{l-1}, x_l)), \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} R(x_{l-1},x_l) = max\{\psi_{cp}(x_{l-1},x_l), \tfrac{\psi_{cp}(x_{l-1},gx_{l-1})\psi_{cp}(x_l,gx_{l-1})}{1+(\psi_{cp}(x_{l-1},x_l))^2}, \\ \tfrac{\psi_{cp}(x_{l-1},gx_l)\psi_{cp}(x_l,gx_l)}{1+\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,gx_l)}\} \leq \\ \leq max\{\psi_{cp}(x_{l-1},x_l), \tfrac{\psi_{cp}(x_{l-1},x_l)\psi_{cp}(x_l,x_l)}{1+(\psi_{cp}(x_{l-1},x_l))^2}, \\ \tfrac{\psi_{cp}(x_{l-1},x_{l+1})\psi_{cp}(x_l,x_{l+1})}{1+\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})}\} \leq \\ \leq max\{\psi_{cp}(x_{l-1},x_l), \tfrac{\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})\psi_{cp}(x_l,x_{l+1})}{1+\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})}\}. \end{aligned}$$

Therefore,

$$R(x_{l-1},x_l) \leq max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}\,. \tag{3.3}$$

Since $|1+\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})| > |\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})|\,.$

Now, let us take,

$$S(x_{l-1},x_l) = min\{\frac{\psi_{cp}(x_{l-1},gx_l)\psi_{cp}(x_l,gx_{l-1})}{1+\psi_{cp}(x_{l-1},x_l)}, \frac{\psi_{cp}(x_{l-1},gx_l)\psi_{cp}(x_l,gx_{l-1})}{1+\psi_{cp}(x_{l-1},x_l)}\}$$

$$\leq min\{\frac{\psi_{cp}(x_{l-1},x_{l+1})\psi_{cp}(x_l,x_l)}{1+\psi_{cp}(x_{l-1},x_l)}, \frac{\psi_{cp}(x_{l-1},x_{l+1})\psi_{cp}(x_l,x_l)}{1+\psi_{cp}(x_{l-1},x_l)}\} = 0\,. \tag{3.4}$$

From (3.2), (3.3), (3.4) and let κ and μ we obtain,

$$\begin{aligned} \kappa(\psi_{cp}(x_l,x_{l+1})) \leq \kappa(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\})- \\ -\mu(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}) \leq \\ \leq \kappa(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}) \end{aligned}$$

$$\kappa(\psi_{cp}(x_l,x_{l+1})) \leq \kappa(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}) \tag{3.5}$$

Suppose $max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\} = \psi_{cp}(x_l,x_{l+1})\,.$

Then (3.5) becomes,

$\kappa(\psi_{cp}(x_l,x_{l+1})) \leq \kappa(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}) < \kappa(\psi_{cp}(x_l,x_{l+1}))$

which is the contradiction.

Therefore, $max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\} = \psi_{cp}(x_{l-1},x_l)$. Now,

$$\kappa(\psi_{cp}(x_l,x_{l+1})) \leq \kappa(\psi_{cp}(x_{l-1},x_l)) - \mu(\psi_{cp}(x_{l-1},x_l)) < \kappa(\psi_{cp}(x_{l-1},x_l))\,. \tag{3.6}$$

Since κ is a increasing mapping, therefore$\{\psi_{cp}(x_l,x_{l+1}) : l \in N \cup \{0\}\}$is an increasing sequence of positive numbers, there exists $n \geq 0$ such that $\lim_{l\to\infty} \psi_{cp}(x_l,x_{l+1}) = n$. Let $l \to \infty$ in (3.6), we get $\kappa(n) \leq \kappa(n) - \mu(n) \leq \kappa(n)$.

Therefore, $\mu(n) = 0$. thus $n = 0$.
Hence we have

$$\lim_{l\to\infty} \psi_{cp}(x_l, x_{l+1}) = 0. \tag{3.7}$$

To show that $\{x_l\}$ is a Cauchy sequence in X, let suppose, $\{x_l\}$ is not a Cauchy sequence. Then there exists $\rho > 0$ and two subsequences $\{x_{k(i)}\}$ and $\{x_{l(i)}\}$ such that: $\psi_{cp}(x_{k(i)}, x_{l(i)}) \geq \rho$, $\quad l(i) > k(i) > i$. This shows that $\psi_{cp}(x_{k(i)}, x_{l(i)-1}) < \rho$.
Therefore we get,

$$\begin{aligned}
\rho &\leq \psi_{cp}(x_{k(i)}, x_{l(i)}) \\
&\leq \psi_{cp}(x_{k(i)}, x_{k(i)-1}) + \psi_{cp}(x_{k(i)-1}, x_{l(i)}) \\
&\leq \psi_{cp}(x_{k(i)}, x_{k(i)-1}) + \psi_{cp}(x_{k(i)-1}, x_{l(i)-1}) + \psi_{cp}(x_{l(i)-1}, x_{l(i)}) \\
&\leq 2\psi_{cp}(x_{k(i)}, x_{k(i)-1}) + \psi_{cp}(x_{k(i)}, x_{l(i)-1}) + \psi_{cp}(x_{l(i)-1}, x_{l(i)}) \\
&< 2\psi_{cp}(x_{k(i)}, x_{k(i)-1}) + \rho + \psi_{cp}(x_{l(i)-1}, x_{l(i)}) .
\end{aligned}$$

Let $i \to \infty$ in the equation (3.7) and we obtain:

$$\begin{aligned}
\lim_{l\to\infty} \psi_{cp}(x_{k(i)}, x_{l(i)}) &= \lim_{l\to\infty} \psi_{cp}(x_{k(i)-1}, x_{l(i)}) \\
&= \psi_{cp}(x_{k(i)}, x_{l(i)-1}) \\
&= \psi_{cp}(x_{k(i)-1}, x_{l(i)-1}) \\
&= \rho .
\end{aligned}$$

From (κ, μ) rational contraction mapping we have,

$$\begin{aligned}
\kappa(\psi_{cp}(x_{k(i)}, x_{l(i)})) &= \kappa(\psi_{cp}(gx_{k(i)-1}, gx_{l(i)} - 1)) \\
&\leq \kappa(R(x_{k(i)-1}, x_{l(i)-1})) - \mu(R(x_{k(i)-1}, x_{l(i)-1})) \\
&\quad +M\kappa(S(x_{k(i)-1}, x_{l(i)-1})) ,
\end{aligned}$$

where

$$\begin{aligned}
R(x_{k(i)-1}, x_{l(i)-1}) = max\{&(\psi_{cp}(x_{k(i)-1}, x_{l(i)-1}), \\
&\tfrac{\psi_{cp}(x_{k(i)-1}, gx_{k(i)-1})\psi_{cp}(x_{l(i)-1}, gx_{k(i)-1})}{1+(\psi_{cp}(x_{k(i)-1}, x_{l(i)-1}))^2}, \\
&\tfrac{\psi_{cp}(x_{k(i)-1}, gx_{l(i)-1})\psi_{cp}(x_{l(i)-1}, gx_{l(i)-1})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})+\psi_{cp}(x_{l(i)-1}, gx_{l(i)-1})}\}
\end{aligned}$$

$$\begin{aligned}
= max\{&(\psi_{cp}(x_{k(i)-1}, x_{l(i)-1}), \frac{\psi_{cp}(x_{k(i)-1}, x_{k(i)})\psi_{cp}(x_{l(i)-1}, x_{k(i)})}{1 + (\psi_{cp}(x_{k(i)-1}, x_{l(i)-1}))^2}, \\
&\frac{\psi_{cp}(x_{k(i)-1}, x_{l(i)})\psi_{cp}(x_{l(i)-1}, x_{l(i)})}{1 + \psi_{cp}(x_{k(i)-1}, x_{l(i)-1}) + \psi_{cp}(x_{l(i)-1}, x_{l(i)})}\}
\end{aligned} \tag{3.8}$$

$$S(x_{k(i)-1}, x_{l(i)-1}) = min\{\frac{\psi_{cp}(x_{k(i)-1}, gx_{k(i)-1})\psi_{cp}(x_{l(i)-1}, gx_{k(i)-1})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})},$$
$$\frac{\psi_{cp}(x_{k(i)-1}, gx_{l(i)-1})\psi_{cp}(x_{l(i)-1}, gx_{k(i)-1})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})}\}$$

$$= min\{\frac{\psi_{cp}(x_{k(i)-1}, x_{k(i)})\psi_{cp}(x_{l(i)-1}, x_{k(i)})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})},$$
$$\frac{\psi_{cp}(x_{k(i)-1}, x_{l(i)})\psi_{cp}(x_{l(i)-1}, x_{k(i)})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})}\}. \tag{3.9}$$

let $i \to \infty$ in (3.9). Therefore

$$\lim_{i\to\infty} R(x_{k(i)-1}, x_{l(i)-1}) = \rho$$
$$\lim_{i\to\infty} S(x_{k(i)-1}, x_{l(i)-1}) = \rho.$$

Letting $i \to \infty$ in (3.8) then it becomes: $\kappa(\rho) \leq \kappa(\rho) - \mu(\rho) < \kappa(\rho)$, which is a contradiction. Hence $(x_{l+1} = gx_l)$ is a Cauchy sequence in X. Since X is a complete space find that $v \in X$ such that $\lim_{l\to\infty} x_{l+1} = \lim_{l\to\infty} gx_l = v$.
Let $gx_l \to gv$ since g is a continuous.
Therefore by limit uniqueness we find $fv = v$.
Hence, v is a fixed point of g. □

Without assuming the continuous the theorem 3.1 we have the following fixed point.

Theorem 3.2. *Let $(X, \leq, \psi_{cp})$ be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let $g : X \to X$ be a increasing mapping with respect to $\leq$. Suppose that g is an (κ, μ)- rational contractive mapping for all comparable $x, y \in X$ then g has a fixed point.*

Proof. The same argument followed from the theorem 3.1, we construct an non-decreasing sequence $\{x_l\}$ in X such that $x_l \to v$ for some $v \in X$. It is enough to show that g has a fixed point. By (κ, μ) rational contraction mapping we have,
(3.10)
$$\kappa(\psi_{cp}(x_{l+1}, gv)) = \kappa(\psi_{cp}(gx_l, gv)) \leq \kappa(R(x_l, v)) - \mu(R(x_l, v)) + M\kappa(S(x_l, v))$$

where

$$R(x_l, v) = max\{\psi_{cp}(x_l, v), \frac{\psi_{cp}(x_l,gx_l)\psi_{cp}(v,gx_l)}{1+(\psi_{cp}(x_l,v))^2}, \frac{\psi_{cp}(x_l,gv)\psi_{cp}(v,gv)}{1+\psi_{cp}(x_l,v)+\psi_{cp}(v,gv)}\}$$
$$= max\{\psi_{cp}(x_l, v), \frac{\psi_{cp}(x_l,x_{l+1})\psi_{cp}(v,x_{l+1})}{1+(\psi_{cp}(x_l,v))^2}, \frac{\psi_{cp}(x_l,gv)\psi_{cp}(v,gv)}{1+\psi_{cp}(x_l,v)+\psi_{cp}(v,gv)}\}$$

$$S(x_l, v) = min\{\frac{\psi_{cp}(x_l, gx_l)\psi_{cp}(v, gx_l)}{1 + \psi_{cp}(x_l, v)}, \frac{\psi_{cp}(x_l, gv)\psi_{cp}(v, gx_l)}{1 + \psi_{cp}(x_l, v)}\}$$
$$= min\{\frac{\psi_{cp}(x_l, x_{l+1})\psi_{cp}(v, x_{l+1})}{1 + \psi_{cp}(x_l, v)}, \frac{\psi_{cp}(x_l, gv)\psi_{cp}(v, x_{l+1})}{1 + \psi_{cp}(x_l, v)}\} \tag{3.11}$$

As $l \to \infty$ in (3.10) we obtain $R(x_l, v) \to \psi_{cp}(v, gv)$ and $S(x_l, v) \to 0$. When $l \to \infty$ in (3.11) we obtain $\kappa(\psi_{cp}(v, gv)) \leq \kappa(\psi_{cp}(v, gv)) - \mu(\psi_{cp}(v, gv))$ so, $(\psi_{cp}(v, gv)) = 0$. Therefore $v = gv$. Thus v is a fixed point of g. □

Corollary 3.1. *Let $(X, \leq, \psi_{cp})$ be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let $g : X \to X$ be a increasing continuous mapping with respect to $\leq$. Suppose that $b \in [0, 1)$ and $M \geq 0$ such that*

$$\psi(gx, gy) \leq bmax\{\psi_{cp}(x, y), \frac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1 + (\psi_{cp}(x, y))^2}, \frac{\psi_{cp}(x, gy)\psi_{cp}(y, gy)}{1 + \psi_{cp}(x, y) + \psi_{cp}(y, gy)}\}$$
$$+Mmin\{\frac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1 + \psi_{cp}(x, y)}, \frac{\psi_{cp}(x, gy)\psi_{cp}(y, gx)}{1 + \psi_{cp}(x, y)}\}$$

for all comparable $x, y \in X$ then g has a fixed point.

Proof. From the theorem 3.1 let us consider $\kappa(a) = a$ and $\mu(a) = (1 - b)a$ for every $a \in [0, \infty]$. Hence it shows that g has a fixed point. □

Without assuming continuity of g in the corollary 3.1.

Corollary 3.2. *Let $(X, \leq, \psi_{cp})$ be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let $g : X \to X$ be a increasing mapping with respect to $\leq$. Suppose that $b \in [0, 1)$ and $M \geq 0$ such that*

$$\psi(gx, gy) \leq bmax\{\psi_{cp}(x, y), \frac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1 + (\psi_{cp}(x, y))^2}, \frac{\psi_{cp}(x, gy)\psi_{cp}(y, gy)}{1 + \psi_{cp}(x, y) + \psi_{cp}(y, gy)}\}$$
$$+Mmin\{\frac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1 + \psi_{cp}(x, y)}, \frac{\psi_{cp}(x, gy)\psi_{cp}(y, gx)}{1 + \psi_{cp}(x, y)}\}$$

for all comparable $x, y \in X$ then g has a fixed point.

Proof. It follows from the theorem 3.2. Let us consider $\kappa(a) = a$ and $\mu(a) = (1-b)a$ for every $a \in [0, \infty]$. Hence it shows that g has a fixed point. □

Example 1. *Consider* $X = \{0, 1, 2, 3.....\}$ *Define the mapping* $g : X \to X$ *defined by:*

$$gx = \begin{cases} 0, & x = 0. \\ x - 3, & x \neq 0. \end{cases}$$

$$gy = \begin{cases} 0, & x \in \{0, 1, 2\}. \\ x - 5, & x \geq 3. \end{cases}$$

Define $\psi_{cp} : X \times X \to C$ *such that*

$$\psi_{cp} = \begin{cases} 0, & x = y. \\ x + 2y, & x \neq y. \end{cases}$$

Then (κ, μ) *rational contraction mapping has a fixed point.*

4. Applications

Let ζ be the set of mapping $\mu : [0, \infty) \to [0, \infty)$ satisfying the hypotheses

(i) Every $\mu \in \zeta$ is a Lebesgue integrable on each compact subset of $[0, \infty)$
(ii) For all $\mu \in \zeta$ and $\rho > 0$

$$\int_0^{\rho} \mu(e)de > 0.$$

Let the function $\kappa : [0, \infty) \to [0, \infty)$ be defined by

$$\kappa(w) = \int_0^{w} \mu(e)de > 0,$$

is an altering distance function. It is obvious to check the function. Now the results follows

Corollary 4.1. *Let* $(X, \leq, \psi_{cp})$ *be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let* $g : X \to X$ *be a increasing continuous* *mapping with respect to $\leq$. Suppose that $b \in [0,1)$ and $M \geq 0$ such that*

$$\int_0^{\psi_{cp}(gx,gy)} \mu(e)de \leq b \int_0^{max\{\psi_{cp}(x,y), \frac{\psi_{cp}(x,gx)\psi_{cp}(y,gx)}{1+(\psi_{cp}(x,y))^2}, \frac{\psi_{cp}(x,gy)\psi_{cp}(y,gy)}{1+\psi_{cp}(x,y)+\psi_{cp}(y,gy)}\}} \mu(e)de$$
$$+M \int_0^{min\{\frac{\psi_{cp}(x,gx)\psi_{cp}(y,gx)}{1+\psi_{cp}(x,y)}, \frac{\psi_{cp}(x,gy)\psi_{cp}(y,gx)}{1+\psi_{cp}(x,y)}\}} \mu(e)de$$

for all comparable $x, y \in X$ then g has a fixed point.

Proof. It follows from the corollary 3.1 by taking

$$\kappa(w) = \int_0^w \mu(e)de.$$

□

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Numerical Functional Analysis and Optimization ›

Volume 41, 2020 - Issue 9

Articles

Edelstein's Theorem for Cyclic Contractive Mappings in Strictly Convex Banach Spaces

M. Gabeleh, J. Maria Felicit & A. Anthony Eldred

Pages 1027-1044 | Received 30 Jan 2019, Accepted 26 Feb 2020, Published online: 09 Mar 2020

Download citation https://doi.org/10.1080/01630563.2020.1737114 Check for updates

Mathematical Modelling and Analysis
Volume 25, Issue 4, 661–679, 2020
https://doi.org/10.3846/mma.2020.10517

http://mma.vgtu.lt
ISSN: 1392-6292
eISSN: 1648-3510

An Overlapping Schwarz Method for Singularly Perturbed Fourth-Order Convection-Diffusion Type

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Received June 13, 2019; revised August 31, 2020; accepted August 31, 2020

Abstract. In this paper, we have constructed an iterative numerical method based on an overlapping Schwarz procedure with uniform mesh for singularly perturbed fourth-order of convection-diffusion type. The method splits the original domain into two overlapping subdomains. A hybrid difference scheme is proposed in which on the boundary layer region we use the central finite difference scheme on a uniform mesh while on the non-layer region we use the mid-point difference scheme on a uniform mesh. It is shown that the method produces numerical approximations which converge in the maximum norm to the exact solution. We prove that, when appropriate subdomains are used the method produces convergence of almost second-order. Furthermore, it is shown that, two iterations are sufficient to achieve the expected accuracy. Numerical examples are presented to support the theoretical results.

Keywords: singularly perturbed problems, convection-diffusion equations, Schwarz method, hybrid difference scheme.

AMS Subject Classification: 65L10.

1 Introduction

Singular Perturbation Problems (SPPs) appear in many branches of applied mathematics, like fluid dynamics, quantum mechanics, turbulent interaction of waves and currents, electrodes theory, etc. The convergence of the numerical approximations generated by standard numerical methods applied to such problems depends adversely on the singular perturbation parameter. Most of these works have concentrated on second-order single differential equations ([4] and the references therein), but for fourth-order equations only few results are reported in the literature [2, 15, 16, 17].

Numerical methods for singularly perturbed problems comprising domain decomposition and Schwarz iterative techniques have been examined by various authors, for example, in [1, 6, 7, 8, 9, 10, 18, 20]. In [10], the authors examined a continuous overlapping Schwarz method for a singularly perturbed convection-diffusion equation with arbitrary fixed interface positions and found it to be uniformly convergent with respect to the perturbation parameter. In [20], an analysis of overlapping domain decomposition methods for singularly perturbed reaction-diffusion problems with distinct small positive parameters is presented. The authors of [20] found a flaw in the analysis of domain decomposition methods explored in [6, 13, 18]. The authors observation is that the constant C is not independent of the iteration number k and it is growing at each induction step in their proof of [6, 13, 18]. But in [20] the authors have presented an alternate analysis of overlapping domain decomposition methods for singularly perturbed reaction-diffusion problems with two parameters and problems in [18].

The authours of [8,9] have concluded that the numerical solution of classical finite difference scheme used in Schwarz method does not converge to the exact solution of their test problem which is a single equation. But our proposed scheme used in Schwarz method [3] has overcome the fundamental difficulty mentioned by the authours of [8,9]. In [8,9], the authors used the same scheme in both the layer and non-layer regions, whereas in our case we used different schemes in each region.

As far as the authors knowledge goes fourth-order SPPs have not yet been examined for higher-order of convergence. Therefore, we are interested in constructing a numerical method for fourth-order SPPs. Of primary interest we have been proved that when appropriate subdomains are used the method produce convergence of almost second-order.

Motivated by the works of [2,10,15,16,17] we have examined experimentally the performance of Schwarz method to the fourth-order Singularly Perturbed Boundary Value Problems (SPBVPs) described as below.

$$-\varepsilon y^{iv}(x) + a(x)y'''(x) + b(x)y''(x) - c(x)y(x) = -f(x), \quad x \in \Omega, \tag{1.1}$$

$$y(0) = q_1, \quad y''(0) = -q_2, \quad y(1) = q_3 \quad y''(1) = -q_4, \tag{1.2}$$

where $a(x), b(x),\ c(x)$ are sufficiently smooth functions satisfying the following conditions:

$$a(x) \geq \alpha, \quad \alpha > 1, \tag{1.3}$$

$$b(x) \geq 0,$$

$$0 \geq c(x) \geq -\gamma, \quad \gamma > 0, \tag{1.4}$$

$$\alpha > 5\gamma \tag{1.5}$$

with $0 < \varepsilon \ll 1$, $\Omega = (0, 1)$, $\bar{\Omega} = [0, 1]$ and $y \in C^{(4)}(\Omega) \cap C^{(2)}(\bar{\Omega})$, which have important applications in fluid dynamics, have been studied in [5], and the references therein. The SPBVPs (1.1)–(1.2) can be transformed into an equivalent weakly coupled system of two ODEs subject to suitable boundary conditions of the form:

$$\begin{cases} L_1\boldsymbol{y}(x) \equiv -y_1''(x) - y_2(x) = 0, \quad x \in \Omega, \\ L_2\boldsymbol{y}(x) \equiv -\varepsilon y_2''(x) + a(x)y_2'(x) \\ \qquad\qquad +b(x)y_2(x) + c(x)y_1(x) = f(x), \quad x \in \Omega, \end{cases} \tag{1.6}$$

$$y_1(0) = q_1, \quad y_2(0) = q_2, \quad y_1(1) = q_3, \quad y_2(1) = q_4, \tag{1.7}$$

where $\boldsymbol{y} = (y_1, y_2)^T$ and $a(x),\ b(x), c(x)$ are sufficiently smooth functions satisfying (1.3)–(1.5). The above weakly coupled system can be written in the matrix-vector form as

$$\mathbf{L}\boldsymbol{y} \equiv \begin{pmatrix} L_1\boldsymbol{y} \\ L_2\boldsymbol{y} \end{pmatrix} \equiv \begin{pmatrix} -\frac{d^2}{dx^2} & 0 \\ 0 & -\varepsilon\frac{d^2}{dx^2} \end{pmatrix} \boldsymbol{y} + \mathbf{A}(x)\boldsymbol{y}' + \mathbf{B}(x)\boldsymbol{y} = \mathbf{f}(x),\ x \in \Omega,$$

$$\boldsymbol{y}(0) = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \quad \boldsymbol{y}(1) = \begin{pmatrix} q_3 \\ q_4 \end{pmatrix},$$

where $\boldsymbol{y}(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$, $\mathbf{f}(x) = \begin{pmatrix} 0 \\ f(x) \end{pmatrix}$, $\mathbf{A}(x) = \begin{pmatrix} 0 & 0 \\ 0 & a(x) \end{pmatrix}$ and $\mathbf{B}(x) = \begin{pmatrix} 0 & -1 \\ c(x) & b(x) \end{pmatrix}$. Let $\beta = \min\{-1, b(x) + c(x)\}$.

In this paper, of primary interest we have proved that discrete Schwarz method converge to the solution of the continuous problem. The method is shown to be of almost second-order convergence. Furthermore, we show that, just two iterations are required to achieve the expected accuracy.

Remark 1. The solution of the problem (1.1)–(1.2) exhibits a boundary layer at $x = 1$ which is less severe because the boundary conditions are prescribed for the derivative of the solution [14]. The condition (1.3) says that (1.1)–(1.2) is a non-turning point problem. The condition (1.4) is known as the quasi-monotonicity condition [14]. The maximum principle theorem for the above system (1.1)–(1.2) and for the corresponding discrete problem are established using the conditions (1.3)–(1.4) and using this principle, we can establish a stability result.

The outline of rest of the paper is as follows. In Section 2, the continuous Schwarz method is described. The derivative estimates are obtained in Section 3. In Section 4, the discrete Schwarz method is described. The maximum pointwise error bounds are obtained in Section 5. Numerical experiments are presented in Section 6 and finally, conclusions are included in Section 7.

Notations: Throughout the paper we use C, with or without subscript to denote a generic positive constant independent of ε, the iteration k and the discretization parameter N.

Let $\boldsymbol{y} : D \to \mathbb{R}$, $D \subseteq \mathbb{R}$. The appropriate norm for studying the convergence of the numerical solution to the exact solution of a SPP is $\|y\|_D = \sup_{x \in D} |y(x)|$. For a vector $\boldsymbol{y} = (y_1, y_2)^T$, we define $\|\boldsymbol{y}\| = \max_{j=1,2} |y_j|$.

For a vector valued function $\boldsymbol{z} = (z_1, z_2)^T$, define $\|\boldsymbol{z}\|_\Omega = \max\{\|z_1\|_\Omega, \|z_2\|_\Omega\}$. Given any two vector valued functions, $\boldsymbol{z}$ and $\boldsymbol{y}$, $\boldsymbol{z} \geq \boldsymbol{y}$ if $z_j \geq y_j$ for all $j = 1, 2$. For a vector of mesh functions $\mathbf{Z}(x_i) = (Z_1(x_i), Z_2(x_i))^T$, define

$$\|\boldsymbol{Z}\|_{\Omega^N} = \max_{j=1,2} \left(\max_{x_i \in \Omega^N} |Z_j(x_i)| \right).$$

2 Continuous Schwarz method

In this section, a continuous Schwarz method is described. This process generates a sequence of iterates $\{\boldsymbol{y}^{[k]}\}$, which converges as $k \to \infty$ to the exact solution $\boldsymbol{y}$. Further we prove the maximum principle for (1.6)–(1.7). Using this principle, a stability result is stated. First, we split the domain into two overlapping subdomains as $\Omega_c = (0, 1-\tau)$ and $\Omega_r = (1-2\tau, 1)$, where the subdomain transition parameter is an appropriate constant, defined in Section 4. The iterative process is defined as follows:

$$\boldsymbol{y}^{[0]}(x) \equiv 0, \quad 0 < x < 1, \qquad \boldsymbol{y}^{[0]}(0) = \boldsymbol{y}(0), \quad \boldsymbol{y}^{[0]}(1) = \boldsymbol{y}(1).$$

For $k \geq 1$, the iterates $\boldsymbol{y}^{[k]}(x)$ are defined by

$$\boldsymbol{y}^{[k]}(x) = \begin{cases} \boldsymbol{y}_c^{[k]}(x) & \text{for} \quad x \in \bar{\Omega}_c, \\ \boldsymbol{y}_r^{[k]}(x) & \text{for} \quad x \in \bar{\Omega}_r \backslash \bar{\Omega}_c, \end{cases}$$

where $\boldsymbol{y}_p^{[k]}, \quad p = \{c, r\}$ are the solutions of the problems

$$\begin{array}{rcl} \mathbf{L}\boldsymbol{y}_r^{[k]}(x) & = & \mathbf{f} \ \text{in} \ \Omega_r, \quad \boldsymbol{y}_r^{[k]}(1-2\tau) = \boldsymbol{y}^{[k-1]}(1-2\tau), \quad \boldsymbol{y}_r^{[k]}(1) = \boldsymbol{y}(1) \\ \mathbf{L}\boldsymbol{y}_c^{[k]}(x) & = & \mathbf{f} \ \text{in} \ \Omega_c, \quad \boldsymbol{y}_c^{[k]}(0) = \boldsymbol{y}(0), \quad \boldsymbol{y}_c^{[k]}(1-\tau) = \boldsymbol{y}_r^{[k]}(1-\tau). \end{array}$$

Letting $\Omega_p = (d, e), \quad \bar{\Omega}_p = [d, e], \quad p = \{c, r\}$, note that the BVPs (1.6)–(1.7) satisfies the following maximum principle on each $\bar{\Omega}_p$.

Theorem 1. (Maximum principle). *Consider the BVPs (1.6)–(1.7). Let* $y_1(d) \geq 0, \quad y_2(d) \geq 0$, *and* $y_1(e) \geq 0$ *and* $y_2(e) \geq 0$, $L_1\boldsymbol{y}(x) \geq 0$, *and* $L_2\boldsymbol{y}(x) \geq 0$, *for* $x \in \Omega_p$. *Then,* $\boldsymbol{y}(x) \geq 0$, $\forall x \in \bar{\Omega}_p$.

Proof. Define the test functions $\boldsymbol{s}(x) = (s_1(x), s_2(x))^T$ by

$$s_1(x) = 5 - x^2, \quad s_2(x) = 1 + x, \quad x \in \bar{\Omega}_p.$$

Clearly, $s_1(d) > 0, \quad s_2(d) > 0, \ s_1(e) > 0, \quad s_2(e) > 0$. We can easily prove that $L_1\boldsymbol{s}(x) > 0$ and $L_2\boldsymbol{s}(x) > 0$, for $x \in \Omega_p$. Assume that the theorem is not true. We define

$$\xi = \max\Big\{ \max_{x \in \bar{\Omega}_p} \big(-y_1/s_1\big)(x), \ \max_{x \in \bar{\Omega}_p} \big(-y_2/s_2\big)(x) \Big\}.$$

Then, $\xi > 0$. Also, $(y_1 + \xi s_1)(x) \geq 0$, $(y_2 + \xi s_2)(x) \geq 0$, $\forall x \in \bar{\Omega}_p$. Furthermore, there exists a point $x_0 \in \Omega_p$ such that either

$$(y_1 + \xi s_1)(x_0) = 0 \quad \text{or} \quad (y_2 + \xi s_2)(x_0) = 0 \quad \text{or both.}$$

Case 1: $(y_1+\xi s_1)(x_0)=0$, for $x_0 \in \Omega_p$. This implies that $y_1+\xi s_1$ attains its minimum at $x=x_0$. Therefore,

$$0 < L_1(\boldsymbol{y}+\xi \boldsymbol{s})(x_0) = -(y_1+\xi s_1)^{''}(x_0) - (y_2+\xi s_2)(x_0) \le 0,$$

which is a contradiction.

Case 2: $(y_2+\xi s_2)(x_0)=0$, for $x_0 \in \Omega_p$. This implies that $y_2+\xi s_2$ attains its minimum at $x=x_0$. Therefore,

$$\begin{aligned} 0 < L_2(\boldsymbol{y}+\xi \boldsymbol{s})(x_0) &= -\varepsilon(y_2+\xi s_2)^{''}(x_0) + a(x)(y_2+\xi s_2)'(x_0) \\ &\quad + b(x)(y_2+\xi s_2)(x_0) + c(x)(y_1+\xi s_1)(x_0) \le 0, \end{aligned}$$

which is a contradiction. Hence it can be conclude that $\boldsymbol{y}(x) \ge 0$, $\forall x \in \bar{\Omega}$. $\square$

An immediate consequence of this is the following stability result.

Lemma 1. (Stability result). *If $\boldsymbol{y}(x)$ is the solution of the BVPs (1.6)–(1.7) then $\forall x \in \bar{\Omega}_p$*

$$\|\boldsymbol{y}\| \le C \max\{|y_1(d)|,\, |y_2(d)|,\, |y_1(e)|,\, |y_2(e)|,\, \max_{x\in\Omega_p} |L_1\boldsymbol{y}(x)|,\, \max_{x\in\Omega_p} |L_2\boldsymbol{y}(x)|\}.$$

Proof. Set

$$M = C \max\{|y_1(d)|,\, |y_2(d)|,\, |y_1(e)|,\, |y_2(e)|,\, \max_{x\in\Omega_p} |L_1\boldsymbol{y}(x)|,\, \max_{x\in\Omega_p} |L_2\boldsymbol{y}(x)|\}.$$

Define two barrier functions $\boldsymbol{w}^{\pm}(x) = (w_1^{\pm}(x),\, w_2^{\pm}(x))^T$ by

$$w_1^{\pm}(x) = M(5-x^2) \pm y_1(x) \quad \text{and} \quad w_2^{\pm}(x) = M(1+x).$$

For $x \in \Omega_c$, we have

$$\begin{aligned} L_1\boldsymbol{w}^{\pm}(x) &= -w_1^{\pm\prime\prime}(x) - w_2^{\pm}(x) > M\tau \pm L_1\boldsymbol{y}(x) \ge 0, \\ L_2\boldsymbol{w}^{\pm}(x) &= -\varepsilon w_2^{\pm\prime\prime}(x) + a(x)w_2^{\pm\prime}(x) + b(x)w_2^{\pm}(x) + c(x)w_1^{\pm}(x), \\ &> M(\alpha - 5\gamma) \pm L_2\boldsymbol{y}(x) \ge 0, \end{aligned}$$

by a proper choice of C. For $x \in \Omega_r$, we have

$$\begin{aligned} L_1\boldsymbol{w}^{\pm}(x) &= -w_1^{\pm\prime\prime}(x) - w_2^{\pm}(x) = M(1-x) \pm L_1\boldsymbol{y}(x) \ge 0, \\ L_2\boldsymbol{w}^{\pm}(x) &= -\varepsilon w_2^{\pm\prime\prime}(x) + a(x)w_2^{\pm\prime}(x) + b(x)w_2^{\pm}(x) + c(x)w_1^{\pm}(x), \\ &> M(\alpha - 5\gamma) \pm L_2\boldsymbol{y}(x) \ge 0, \end{aligned}$$

by a proper choice of C. Furthermore, we have

$$\begin{aligned} &w_1^{\pm}(d) = w_1^{\pm}(0) = 5M \pm y_1(0) \ge 0, \quad w_2^{\pm}(d) = w_2^{\pm}(0) = M \pm y_2(0) \ge 0, \\ &w_1^{\pm}(e) = w_1^{\pm}(1-\tau) > 3M \pm y_1(1-\tau) \ge 0, \\ &w_2^{\pm}(e) = w_2^{\pm}(1-\tau) > M \pm y_2(1-\tau) \ge 0, \\ &w_1^{\pm}(d) = w_1^{\pm}(1-2\tau) > 4M \pm y_1(1-2\tau) \ge 0, \\ &w_2^{\pm}(d) = w_2^{\pm}(1-2\tau) > M \pm y_2(1-2\tau) \ge 0, \\ &w_1^{\pm}(e) = w_1^{\pm}(1) = 4M \pm y_1(1) \ge 0, \quad w_2^{\pm}(e) = w_2^{\pm}(1) = 2M \pm y_2(1) \ge 0 \end{aligned}$$

by a proper choice of C. Applying Theorem 1 to the barrier functions $\boldsymbol{w}^{\pm}(x)$, we get the desired result. $\square$

3 Estimates of derivatives

In Section 5 we establish the convergence of the discrete Schwarz method described in Section 4. To prove convergence of the numerical solution, we need the following stronger results on the estimates of the derivatives of the components of the solution of the BVPs (1.6)–(1.7). Now, decompose the solution $\boldsymbol{y}(x)$ of (1.6)–(1.7) into smooth and singular components $\boldsymbol{v}(x)$ and $\boldsymbol{w}(x)$ respectively as

$$\boldsymbol{y}(x) = \boldsymbol{v}(x) + \boldsymbol{w}(x), \tag{3.1}$$

where $\boldsymbol{v}(x) = (v_1(x), v_2(x))^T$ is the solution of the reduced problem of the BVPs (1.6)–(1.7) given by

$$\begin{cases} -v_1''(x) - v_2(x) = 0, \\ a(x)v_2'(x) + b(x)v_2(x) + c(x)v_1(x) = f(x), \\ v_1(0) = q_1, \quad v_1(1) = q_3, \quad v_2(0) = q_2 \end{cases} \tag{3.2}$$

and $\boldsymbol{w}(x) = (w_1(x), w_2(x))^T$ is a layer correction term given by

$$\begin{cases} w_1(x) = -(\varepsilon/a(0))^2(q_4 - v_2(1))e^{-a(0)(1-x)/\varepsilon}, \\ w_2(x) = (q_4 - v_2(1))e^{-a(0)(1-x)/\varepsilon} \end{cases}$$

and $\boldsymbol{w(x)}$ satisfies

$$\begin{cases} -w_1''(x) - w_2(x) = 0, \\ -\varepsilon w_2''(x) + a(0)w_2'(x) = 0, \\ w_1(0) = w_1(1)e^{-a(0)/\varepsilon}, \quad w_1(1) = -w_2(1)(\varepsilon/a(0))^2, \\ w_2(0) = w_2(1)e^{-a(0)/\varepsilon}, \quad w_2(1) = q_4 - v_2(1). \end{cases} \tag{3.3}$$

The following lemma gives estimates of the derivatives of these components.

Lemma 2. *The solution $\boldsymbol{y}(x)$ of the BVPs (1.6)–(1.7) has the decomposition $\boldsymbol{y}(x) = \boldsymbol{v}(x) + \boldsymbol{w}(x)$ into smooth and singular components, satisfy*

$$|\boldsymbol{v}_1^{(l)}(x)| \le C, \quad |\boldsymbol{v}_2^{(l)}(x)| \le C,$$
$$|\boldsymbol{w}_1^{(l)}(x)| \le C\varepsilon^{-(l-2)}e^{-\alpha(1-x)/\varepsilon}, \quad |\boldsymbol{w}_2^{(l)}(x)| \le C\varepsilon^{-(l)}e^{-\alpha(1-x)/\varepsilon},$$

for $0 \le l \le 4$, $\forall x \in \bar{\Omega} = (\bar{\Omega}_r \backslash \bar{\Omega}_c) \cup \bar{\Omega}_c$, $\boldsymbol{v}(x)$ and $\boldsymbol{w}(x)$ are given by (3.2)–(3.3).

Proof. It is easy to check that

$$|v_1^{(l)}(x)| \le C \quad \text{and} \quad |v_2^{(l)}(x)| \le C \quad \text{for} \quad x \in \bar{\Omega}$$

as $a(x)$, $b(x)$, $c(x)$, $f(x)$ are sufficiently smooth functions. Differentiating the equation (3.3) l times and using the method of induction one can get

$$\begin{aligned} |w_1^{(l)}(x)| &\le C\varepsilon^{-(l-2)} \exp(-\alpha(1-x)/\varepsilon), \\ |w_2^{(l)}(x)| &\le C\varepsilon^{-(l)} \exp(-\alpha(1-x)/\varepsilon). \end{aligned}$$

□

4 Discrete Schwarz method

The continuous overlapping Schwarz method described in Section 2 is discretized by introducing uniform meshes on each subdomain. The domain $\Omega = (0,1)$ is divided into two overlapping subdomains as $\Omega_c = (0,1-\tau)$ and $\Omega_r = (1-2\tau,1)$. The subdomain transition parameter τ is chosen to be the Shishkin transition point $\tau = \min\left\{\frac{1}{3}, \frac{4\varepsilon}{\alpha}\ln N\right\}$ as in ([10], p.91). In each subdomain, $\Omega_p = (d,e), \quad p=\{c,r\}$, construct a uniform mesh $\bar{\Omega}_p^N = \{d = x_0 < x_1 < x_2 < \cdots < x_n = e\}$ with $h_p = (x_i - x_{i-1})/N = (e-d)/N$.

In the proposed scheme we use the central finite difference scheme with a uniform mesh on the subdomain Ω_r and the mid-point difference scheme with a uniform mesh on the subdomain Ω_c. Then in each subdomain $\Omega_p^N,\ p=\{c,r\}$, the corresponding discretization is,

$$\mathbf{L}^N\mathbf{Y}_c(x_i)=\begin{cases} L_1^N\mathbf{Y}_c(x_i)=-\delta^2 Y_{1,c}(x_i)-\hat{Y}_{2,c}(x_i)=0, \quad i=1,\dots,N-1,\\ L_2^N\mathbf{Y}_c(x_i)=-\varepsilon\delta^2 Y_{2,c}(x_i)+a_{i-1/2}D^-Y_{2,c}(x_i)+c_{i-1/2}\hat{Y}_{1,c}(x_i)\\ \qquad\qquad +b_{i-1/2}\hat{Y}_{2,c}(x_i)=f_{i-1/2}, \quad i=1,\dots,N-1,\end{cases}$$

$$\mathbf{L}^N\mathbf{Y}_r(x_i)=\begin{cases} L_1^N\mathbf{Y}_r(x_i)=-\delta^2 Y_{1,r}(x_i)-Y_{2,r}(x_i)=0, \quad i=1,\dots,N-1,\\ L_2^N\mathbf{Y}_r(x_i)=-\varepsilon\delta^2 Y_{2,r}(x_i)+a_iD^0Y_{2,r}(x_i)+b_iY_{2,r}(x_i)\\ \qquad\qquad +c_iY_{1,r}(x_i)=f_i, \quad i=1,\dots,N-1,\end{cases}$$

where

$$\delta^2 Y_{j,p}(x_i)=\frac{1}{h_p^2}\left(Y_{j,p}(x_{i+1})-2Y_{j,p}(x_i)+Y_{j,p}(x_{i-1})\right),$$

$$D^-Y_{j,c}(x_i)=\frac{Y_{j,c}(x_i)-Y_{j,c}(x_{i-1})}{h_c}, \quad \hat{Y}_{j,c}(x_i)\equiv(Y_{j,c}(x_i)+Y_{j,c}(x_{i-1}))/2,$$

$$D^0Y_{j,r}(x_i)=\frac{Y_{j,r}(x_{i+1})-Y_{j,r}(x_{i-1})}{2h_r},\ a_{i-1/2}\equiv a((x_{i-1}+x_i)/2),\ a_i\equiv a(x_i),$$

similarly for $b_{i-1/2}$, $c_{i-1/2}$, $f_{i-1/2}$, b_i, c_i and f_i, $j=1,2$.

The discrete problem is $\mathbf{L}^N\mathbf{Y}_p(x_i)=\mathbf{f}(x_i)$, where

$$\mathbf{f}(x_i)=\begin{cases}\mathbf{f}_{i-\frac{1}{2}}, & x_i\in\bar{\Omega}_c^N,\\ \mathbf{f}_i, & x_i\in\bar{\Omega}_r^N.\end{cases}$$

Then the algorithm for discrete Schwarz method is defined as follows.

Step1: We choose the initial mesh function

$$\mathbf{Y}^{[0]}(x_i)\equiv 0, \quad 0<x_i<1, \quad \mathbf{Y}^{[0]}(0)=\boldsymbol{y}(0), \quad \mathbf{Y}^{[0]}(1)=\boldsymbol{y}(1).$$

Step2: We compute the mesh functions $\mathbf{Y}_p{}^{[k]}$, $p=\{r,c\}$ which are the solutions of the following discrete problems

$$\mathbf{L}^N\mathbf{Y}_r^{[k]}(x_i)=\mathbf{f}_i,\ \ x_i\in\Omega_r^N,\ \ \mathbf{Y}_r^{[k]}(1-2\tau)=\bar{\mathbf{Y}}^{[k-1]}(1-2\tau),\ \ \mathbf{Y}_r^{[k]}(1)=\boldsymbol{y}(1),$$

$$\mathbf{L}^N\mathbf{Y}_c^{[k]}(x_i)=\mathbf{f}_{i-\frac{1}{2}},\ \ x_i\in\Omega_c^N,\ \ \mathbf{Y}_c^{[k]}(0)=\boldsymbol{y}(0),\ \ \mathbf{Y}_c^{[k]}(1-\tau)=\bar{\mathbf{Y}}_r^{[k]}(1-\tau),$$

where $\bar{\boldsymbol{Y}}^{[k]}$, $k \geq 1$ denotes the piecewise linear interpolant of $\boldsymbol{Y}^{[k]}$ on the mesh $\bar{\Omega}^N := (\bar{\Omega}_r^N \backslash \bar{\Omega}_c) \cup \bar{\Omega}_c^N$.

Step3: We compute the mesh function $\mathbf{Y}^{[k]}$ by combining together the solutions on the subdomains

$$\mathbf{Y}^{[k]}(x_i) = \begin{cases} \mathbf{Y}_c^{[k]}(x_i), & \text{for } \ x_i \in \bar{\Omega}_c^N, \\ \mathbf{Y}_r^{[k]}(x_i), & \text{for } \ x_i \in \bar{\Omega}_r^N \backslash \bar{\Omega}_c. \end{cases}$$

Step4: If the stopping criterion $\|\mathbf{Y}^{[k+1]} - \mathbf{Y}^{[k]}\|_{\bar{\Omega}^N} \leq tol$ is reached, then stop; otherwise go to Step 2. Here *tol* is the user prescribed accuracy. For each $p = \{c, r\}$, the matrix associated with $\mathbf{L}^N$ is M-matrix, and hence it satisfies the following discrete maximum principle.

Lemma 3. ***(Discrete maximum principle)*** *Assume that $\boldsymbol{Y}(x_0) \geq \boldsymbol{0}$ and $\boldsymbol{Y}(x_N) \geq \boldsymbol{0}$, then $\boldsymbol{L}^N \boldsymbol{Y}(x_i) \geq \boldsymbol{0}$, $\forall x_i \in \Omega_p^N$ implies that $\boldsymbol{Y}(x_i) \geq \boldsymbol{0}$, $\forall x_i \in \bar{\Omega}_p^N$.*

Proof. Please refer to [11, 12] and [19]. □

An immediate consequence of this lemma is the following stability result.

Lemma 4. *If $Y_j(x_i)$ is any mesh function then for all $x_i \in \bar{\Omega}_p^N$*

$$|Y_j(x_i)| \leq C \max\{|Y_1(x_0)|, |Y_1(x_N)|, |Y_2(x_0)|, |Y_2(x_N)|, \|L_1^N \boldsymbol{Y}\|_{\Omega_p^N}, \|L_2^N \boldsymbol{Y}\|_{\Omega_p^N}\}$$

for j=1, 2

Proof. Please refer to [11, 12] and [19]. □

5 Error estimates

In this Section, we estimate the error in discrete Schwarz iterates and prove that two iterations are required to attain almost second-order convergence. Following the method of analysis adapted in [18] and [20] we derive error estimates. The analysis proceeds as follows.

Lemma 5. *Let $\boldsymbol{y}$ be the solution of (1.6)–(1.7) and let $\boldsymbol{Y}^{[k]}$ be the k^{th} iterate of the discrete Schwarz method described as in Section 4. Then, there are constants C such that*

$$\| \boldsymbol{Y}^{[k]} - \boldsymbol{y}\|_{\bar{\Omega}^N} \leq C2^{-k} + CN^{-2} \ln^3 N.$$

Proof. At the first iteration $(\mathbf{Y}^{[0]} - \boldsymbol{y})(0) = \mathbf{0}$ and $(\mathbf{Y}^{[0]} - \boldsymbol{y})(1) = \mathbf{0}$. Since $\mathbf{Y}^{[0]}(x_i) = \mathbf{0}$ for $x_i \in \Omega^N := \{x_1 < x_2 < x_3 \cdots < x_{N-1}\}$ we can use Lemma 1 to show that

$$\|\mathbf{Y}^{[0]} - \boldsymbol{y}\|_{\Omega^N} = \|\boldsymbol{y}\|_{\Omega^N} \leq C.$$

Clearly, there are constants C such that

$$\|\mathbf{Y}^{[0]} - \boldsymbol{y}\|_{\bar{\Omega}^N} \leq C2^0 + CN^{-2} \ln^3 N.$$

Thus, the result holds for $k = 0$ and the proof is now completed by induction. Assume that, for an arbitrary integer $k \geq 0$, there exists C such that

$$\|\mathbf{Y}^{[k]} - \boldsymbol{y}\|_{\bar{\Omega}^N} \leq C2^{-k} + CN^{-2}\ln^3 N.$$

Case (i): Error bound estimation on $\bar{\Omega}_r^N$. In the proposed scheme we use the central finite difference scheme on $\bar{\Omega}_r^N$. One can deduce the following truncation error estimate as in [12] on $x_i \in \bar{\Omega}_r^N$ as

$$\|\mathbf{L}^N(\mathbf{Y} - \boldsymbol{y})\|_{\Omega_r^N} \leq \begin{pmatrix} Ch_r^2\|y_1^{(4)}\|_{\Omega_r^N} \\ C\varepsilon h_r^2\|y_2^{(4)}\|_{\Omega_r^N} + Ch_r^2\|y_2^{(3)}\|_{\Omega_r^N} \end{pmatrix}. \tag{5.1}$$

In order to find a bound on $\|\mathbf{L}^N(\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})\|_{\Omega_r^N}$ we must decompose $\boldsymbol{y}$ as in (3.1). Consider

$$\begin{aligned} \|\mathbf{L}^N(\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})\|_{\Omega_r^N} &= \|\mathbf{f} - \mathbf{L}\boldsymbol{y}\|_{\Omega_r^N} = \|(\mathbf{L}^N - \mathbf{L})\boldsymbol{y}\|_{\Omega_r^N} \\ &\leq \|(\mathbf{L}^N - \mathbf{L})\boldsymbol{v}\|_{\Omega_r^N} + \|(\mathbf{L}^N - \mathbf{L})\boldsymbol{w}\|_{\Omega_r^N}. \end{aligned} \tag{5.2}$$

For the first term on the right-hand side of (5.2), we use the local truncation error estimate (5.1), $h_r \leq CN^{-1}$, $\varepsilon \leq CN^{-1}$, and Lemma 2 to get

$$\begin{aligned} \|(\mathbf{L}^N - \mathbf{L})\boldsymbol{v}\|_{\Omega_r^N} &\leq \begin{pmatrix} Ch_r^2\|v_1^{(4)}\|_{\Omega_r^N} \\ C\varepsilon h_r^2\|v_2^{(4)}\|_{\Omega_r^N} + Ch_r^2\|v_2^{(3)}\|_{\Omega_r^N} \end{pmatrix} \\ &\leq \begin{pmatrix} CN^{-2} \\ CN^{-3} + CN^{-2} \end{pmatrix} \leq CN^{-2}. \end{aligned}$$

For the second term on the right-hand side of (5.2), when $\tau = \frac{4\varepsilon}{\alpha}\ln N$, using the local truncation error estimate (5.1), and $h_r \leq C\varepsilon N^{-1}\ln N$, we have

$$\begin{aligned} \|(\mathbf{L}^N - \mathbf{L})\boldsymbol{w}\|_{\Omega_r^N} &\leq \begin{pmatrix} Ch_r^2\|w_1^{(4)}\|_{\Omega_r^N} \\ C\varepsilon h_r^2\|w_2^{(4)}\|_{\Omega_r^N} + Ch_r^2\|w_2^{(3)}\|_{\Omega_r^N} \end{pmatrix} \\ &\leq \begin{pmatrix} Ch_r^2\varepsilon^{-2} \\ Ch_r^2\varepsilon^{-3} \end{pmatrix} \leq C\varepsilon^{-1}N^{-2}\ln^2 N. \end{aligned}$$

Using the above estimates in (5.2), we have

$$\|\mathbf{L}^N(\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})\|_{\Omega_r^N} \leq CN^{-2}\ln^3 N + C\varepsilon^{-1}N^{-2}\ln^2 N$$

for some C. The end point of the subdomain Ω_r^N is $1 - 2\tau$, which is in general is not in $\Omega^N = \{x_1 < x_2 < x_3 < \ldots < x_{N-1}\}$, so we use a piecewise linear interpolant of the previous iterate to determine $\mathbf{Y}_r^{[k+1]}(1 - 2\tau)$. Now, using our inductive argument, we have

$$\begin{aligned} |(\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})(1 - 2\tau)| &= |(\bar{\mathbf{Y}}^{[k]} - \boldsymbol{y})(1 - 2\tau)| = |(\mathbf{Y}^{[k]} - \boldsymbol{y})(1 - 2\tau)| \\ &\leq |(\mathbf{Y}^{[k]} - \bar{\boldsymbol{y}})(1 - 2\tau)| + |(\bar{\boldsymbol{y}} - \boldsymbol{y})(1 - 2\tau)|, \end{aligned} \tag{5.3}$$

where $\bar{\boldsymbol{y}}$ is the piecewise linear interpolant of $\boldsymbol{y}$ using grid points of $\bar{\Omega}_c^N$. For the second term on the right-hand side of (5.3), using solution decomposition $\boldsymbol{y}$ as in (3.1), we get

$$|(\bar{\boldsymbol{y}} - \boldsymbol{y})(1-2\tau)| \leq |(\bar{\boldsymbol{v}} - \boldsymbol{v})(1-2\tau)| + |(\bar{\boldsymbol{w}} - \boldsymbol{w})(1-2\tau)|. \tag{5.4}$$

Note that $(1-2\tau)$ lies in $\bar{\Omega}_c$. For any $\mathbf{z} \in C^2(\bar{\Omega}_c)$, standard argument of piecewise linear interpolant $\bar{\mathbf{z}}$ gives

$$|(\mathbf{z} - \bar{\mathbf{z}})(1-2\tau)| \leq Ch_c^2 \|\mathbf{z}^{(2)}\|_{\bar{\Omega}_c} \quad \text{and} \quad |(\mathbf{z} - \bar{\mathbf{z}})(1-2\tau)| \leq C\|\mathbf{z}\|_{\bar{\Omega}_c}. \tag{5.5}$$

For the first term on the right-hand side of (5.4), we use the first bound of (5.5), $h_c \leq CN^{-1}$, and Lemma 2 to get

$$|(\bar{\boldsymbol{v}} - \boldsymbol{v})(1-2\tau)| \leq Ch_c^2 \|\boldsymbol{v}^{(2)}\|_{\bar{\Omega}_c} \leq CN^{-2}.$$

For the second term on the right-hand side of (5.4), when $\tau = \frac{4\varepsilon}{\alpha} \ln N$, note that the layer function $\boldsymbol{w}$ is monotonically increasing in the region $(1/3, 1-\tau) \subset \bar{\Omega}_c$. Hence using the second bound of (5.5), we have

$$|(\bar{\boldsymbol{w}} - \boldsymbol{w})(1-2\tau)| \leq C\|\boldsymbol{w}\|_{\bar{\Omega}_c}. \tag{5.6}$$

Now, using (5.6) in (5.3) we have

$$\begin{aligned} |(\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})(1-2\tau)| &\leq C2^{-k} + CN^{-2}\ln^3 N + CN^{-2} \\ &\leq C2^{-k} + CN^{-2}\ln^3 N. \end{aligned}$$

Consider the mesh function

$$\begin{aligned} \Psi^{\pm}(x_i) =& \mathbf{C}\left(\frac{3+x_i}{2}\right)2^{-k} + \mathbf{C}(1+x_i)N^{-2}\ln^3 N \\ &+ \mathbf{C}(x_i - (1-2\tau))\varepsilon^{-1}N^{-2}\ln^2 N \pm (\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})(x_i), \end{aligned}$$

where $\mathbf{C}$ is positive constants suitably chosen so that the following are satisfied.

Note that, $\Psi^{\pm}(1-2\tau) > 0$, $\Psi^{\pm}(1) > 0$ and $L^N\Psi^{\pm}(x_i) > 0$. Using the discrete maximum principle for the operator L^N on $\bar{\Omega}_r^N$ we get,

$$\begin{aligned} \|(\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})\|_{\bar{\Omega}_r^N} \leq& C\left(\frac{3+x_i}{2}\right)2^{-k} + C(1+x_i)N^{-2}\ln^3 N \\ &+ C(x_i - (1-2\tau))\varepsilon^{-1}N^{-2}\ln^2 N. \end{aligned}$$

Consequently,

$$\begin{aligned} \|(\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})\|_{\bar{\Omega}_r^N \setminus \bar{\Omega}_c} &\leq 4C\left(\frac{1}{2}\right)2^{-k} + 2CN^{-2}\ln^3 N + 2C\tau\varepsilon^{-1}N^{-2}\ln^2 N \\ &\leq C2^{-(k+1)} + +CN^{-2}\ln^3 N + C\tau\varepsilon^{-1}N^{-2}\ln^2 N. \end{aligned}$$

But since $\tau = \frac{4\varepsilon}{\alpha}\ln N$, this gives

$$\|(\mathbf{Y}_r^{[k+1]} - \boldsymbol{y})\|_{\bar{\Omega}_r^N \setminus \bar{\Omega}_c} \leq C2^{-(k+1)} + CN^{-2}\ln^3 N. \tag{5.7}$$

Case (ii): Error bound estimation on $\bar{\Omega}_c^N$. We use solution decomposition as in Lemma 2 at each point $x_i \in \bar{\Omega}_c^N$, the difference $(\mathbf{Y}_c^{[k+1]} - \boldsymbol{y})$ can be written in the form

$$(\mathbf{Y}_c^{[k+1]} - \boldsymbol{y})(x_i) = (\mathbf{V}_c^{[k+1]} - \boldsymbol{v})(x_i) + (\mathbf{W}_c^{[k+1]} - \boldsymbol{w})(x_i). \tag{5.8}$$

Suppose that $(1-\tau)$ lies in $\bar{\Omega}_r$. For any $\mathbf{z} \in \mathbf{C}^2(\bar{\Omega}_r)$, standard argument of piecewise linear interpolant $\bar{\mathbf{z}}$ gives

$$|(\mathbf{z} - \bar{\mathbf{z}})(1-\tau)| \leq Ch_r^2 \|\mathbf{z}^{(2)}\|_{\bar{\Omega}_r}. \tag{5.9}$$

In the proposed scheme we use the mid-point difference scheme on $\bar{\Omega}_c^N$. One can deduce the following truncation error estimate as in [12] on $x_i \in \bar{\Omega}_c^N$ as

$$\|(\mathbf{L}^N - \mathbf{L})\boldsymbol{y}\|_{\Omega_c^N} \quad \leq \quad \begin{pmatrix} Ch_c^2 \|y_1^{(4)}\|_{\Omega_c^N} + Ch_c^2 \|y_2^{(2)}\|_{\Omega_c^N} \\ C\varepsilon h_c^2 \|y_2^{(4)}\|_{\Omega_c^N} + Ch_c^2 (\|y_2^{(3)}\|_{\Omega_c^N} + \|y_1^{(2)}\|_{\Omega_c^N}) \end{pmatrix}.$$

Subcase (i): For the first term on the right-hand side of (5.8), using the above local truncation error estimate, $h_c \leq CN^{-1}$, $\varepsilon \leq CN^{-1}$ and Lemma 2, we get

$$\begin{aligned} &\|\mathbf{L}^N (\mathbf{V}_c^{(k+1)} - \boldsymbol{v})\|_{\Omega_c^N} = \|\mathbf{f} - \mathbf{L}\boldsymbol{v}\|_{\Omega_c^N} = \|(\mathbf{L}^N - \mathbf{L})\boldsymbol{v}\|_{\Omega_c^N} \\ &\leq \begin{pmatrix} Ch_c^2 \|v_1^{(4)}\|_{\Omega_c^N} + Ch_c^2 \|v_2^{(2)}\|_{\Omega_c^N} \\ C\varepsilon h_c^2 \|v_2^{(4)}\|_{\Omega_c^N} + Ch_c^2 (\|v_2^{(3)}\|_{\Omega_c^N} + \|v_1^{(2)}\|_{\Omega_c^N}) \end{pmatrix} \leq \begin{pmatrix} CN^{-2} \\ CN^{-2} \end{pmatrix} \leq CN^{-2}. \end{aligned}$$

Now, using our inductive argument, the bound of (5.9), $h_r \leq CN^{-1}$, $\varepsilon \leq CN^{-1}$, and Lemma 2, we get

$$\begin{aligned} |(\mathbf{V}_c^{[k+1]} - \boldsymbol{v})(1-\tau)| \quad &= \quad |(\bar{\mathbf{V}}_r^{[k+1]} - \boldsymbol{v})(1-\tau)| = |(\bar{\mathbf{V}} - \boldsymbol{v})(1-\tau)| \\ &\leq \quad Ch_r^2 \|\boldsymbol{v}^{(2)}\|_{\bar{\Omega}_r} \leq CN^{-2}, \end{aligned}$$

where we have used the fact that $(1-\tau)$ is the mesh point of $\bar{\Omega}_r^N$.

Consider the mesh function

$$\Phi^{\pm}(x_i) \quad = \quad \mathbf{C}\left(\frac{x_i}{2(1-\tau)}\right) 2^{-k} + (1+x_i)\mathbf{C}N^{-2} \pm (\mathbf{V}_c^{[k+1]} - \boldsymbol{v})(x_i),$$

where C is positive constants to be choosen suitably, so that the following expressions are satisfied. Note that $\Phi^{\pm}(0) > 0$, $\Phi^{\pm}(1-\tau) > 0$, $L^N \Phi^{\pm}(x_i) > 0$.

We use the discrete maximum principle for the operator L^N on $\bar{\Omega}_c^N$ to get

$$\begin{aligned} \|\mathbf{V}_c^{[k+1]} - \boldsymbol{v}\|_{\bar{\Omega}_c^N} \quad &\leq \quad C\left(\frac{1}{2}\right) 2^{-k} + C(2-\tau)N^{-2} \\ &\leq \quad C2^{-(k+1)} + CN^{-2}. \end{aligned}$$

Subcase (ii): For the second term on the right-hand side of (5.8), when $\tau = \frac{4\varepsilon}{\alpha} \ln N$, using the arguments discussed as in ([11], Lemma 6) for $x_i \in \Omega_c^N$ we get

$$\|\mathbf{W}_c^{[k+1]} - \boldsymbol{w}\|_{\Omega_c^N} \leq CN^{-2}.$$

Now, using error bound for the smooth and layer parts we get

$$\|(\mathbf{Y}_c^{[k+1]} - \boldsymbol{y})\|_{\Omega_c^N} \le C2^{-(k+1)} + CN^{-2}\ln^3 N. \tag{5.10}$$

Combining the error bounds (5.7) and (5.10), we have

$$\|\mathbf{Y}^{[k+1]} - \boldsymbol{y}\|_{\bar{\Omega}^N} \le C2^{-(k+1)} + CN^{-2}\ln^3 N.$$

This completes the proof. □

Now we will show that the discrete Schwarz iterates converge at a higher rate than that suggested by Lemma 5.

Lemma 6. *Let $\mathbf{Y}^{[k]}(x_i)$ be the k^{th} iterate of the discrete Schwarz method described in Section 4. Then there exists some C such that*

$$\|\,\mathbf{Y}^{[k+1]} - \,\mathbf{Y}^{[k]}\|_{\bar{\Omega}^N} \le C\nu^k, \quad \textit{where} \ \ \nu = \left(1 + \frac{\tau\alpha}{2\varepsilon N}\right)^{-N/2} < 1.$$

Furthermore, if $\tau = \frac{4\varepsilon}{\alpha}\ln N$, then $\nu \le 2N^{-1}$.

Proof. At the first iteration $\|\mathbf{Y}^{[0]}\|_{\Omega^N} = \mathbf{0}$. Then clearly

$$\|\mathbf{Y}^{[1]} - \mathbf{Y}^{[0]}\|_{\Omega^N} = \|\mathbf{Y}^{[1]}\|_{\Omega^N}.$$

$\mathbf{Y}_r^{[1]}$ satisfies

$$\mathbf{L}^N\mathbf{Y}_r^{[1]} = \mathbf{f}_i \quad \text{for} \quad x_i \in \Omega_r^N,$$
$$\mathbf{Y}_r^{[1]}(1-2\tau) = \bar{\mathbf{Y}}^{[0]}(1-2\tau), \quad \mathbf{Y}_r^{[1]}(1) = \boldsymbol{y}(1).$$

Therefore, we use Lemma 4 to obtain $\|\mathbf{Y}_r^{[1]}\|_{\bar{\Omega}_r^N} \le C$.

Consequently, $\|\mathbf{Y}_r^{[1]}\|_{\bar{\Omega}_r^N \setminus \bar{\Omega}_c} \le C$. Also $\mathbf{Y}_c^{[1]}$ satisfies

$$\mathbf{L}^N\mathbf{Y}_c^{[1]} = \mathbf{f}_{i-1/2} \ \text{ for } \ x_i \in \Omega_c^N,$$
$$\mathbf{Y}_c^{[1]}(0) = \boldsymbol{y}(0), \quad \mathbf{Y}_c^{[1]}(1-\tau) = \bar{\mathbf{Y}}_r^{[1]}(1-\tau).$$

Therefore, we can apply Lemma 4 to get $\|\mathbf{Y}_c^{[1]}\|_{\bar{\Omega}_c^N} \le C$. Combining all these estimates we obtain

$$\|\mathbf{Y}^{[1]} - \mathbf{Y}^{[0]}\|_{\bar{\Omega}^N} \le C\nu^0.$$

Thus, the result holds for $k = 0$ and the proof is now completed by induction argument. Assume that for an arbitrary integer $k \ge 0$

$$\|\mathbf{Y}^{[k+1]} - \mathbf{Y}^{[k]}\|_{\bar{\Omega}^N} \le \mathbf{C}\nu^k, \quad \text{where} \ \ \nu = \left(1 + \frac{\alpha\tau}{2\varepsilon N}\right)^{-N/2}.$$

Note that $\mathbf{L}^N(\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(x_i) = \mathbf{0}$ for $x_i \in \Omega_c^N$, $(\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(0) = \mathbf{0}$, and $|(\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(1-\tau)| \le C\nu^k$.

Let $\Phi_c^{[k+1]}(x_i) = \left(\Phi_{1,c}^{[k+1]}(x_i), \Phi_{2,c}^{[k+1]}(x_i)\right)^T$ be the solution of

$$\begin{cases} A\delta^2\Phi_c^{[k+1]}(x_i) + \alpha D^- \Phi_c^{[k+1]}(x_i) + \beta\hat{\Phi}_c^{[k+1]}(x_i) = \mathbf{0} \quad \text{for} \quad x_i \in \Omega_c^N, \\ \Phi_c^{[k+1]}(x_0) = \mathbf{0}, \quad \Phi_c^{[k+1]}(x_n) = \mathbf{C}\nu^k, \end{cases} \tag{5.11}$$

where $A = \begin{pmatrix} -1 & 0 \\ 0 & -\varepsilon \end{pmatrix}$. Using the maximum principle argument we note that $\Phi_c^{[k+1]}(0) \geq \mathbf{0}$, $\Phi_c^{[k+1]}(1-\tau) \geq \mathbf{0}$, $\Phi_c^{[k+1]}(x_i) \geq \mathbf{0}$ for $x_i \in \bar{\Omega}_c^N$, and thus one can easily deduce that $L^N\Phi_c^{[k+1]}(x_i) \geq \mathbf{0}$, for $x_i \in \Omega_c^N$. Hence

$$\begin{aligned} &L^N(\Phi_c^{[k+1]} - (\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]}))(x_i) = L^N(\Phi_c^{[k+1]})(x_i) - L^N(\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(x_i), \\ &\qquad \geq \mathbf{0}, \text{ as } L^N(\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(x_i) = \mathbf{0} \text{ for } x_i \in \Omega_c^N, \\ &\Phi_c^{[k+1]}(0) - (\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(0) \geq \mathbf{0},\ \Phi_c^{[k+1]}(1-\tau) - (\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(1-\tau) \geq \mathbf{0}. \end{aligned}$$

Then by using Lemma 3 we have

$$(\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(x_i) \leq \Phi_c^{[k+1]}(x_i) \quad \text{for} \quad x_i \in \Omega_c^N. \tag{5.12}$$

The exact solution to the difference problem (5.11) is

$$\Phi_c^{[k+1]}(x_i) = \mathbf{C}\nu^k(m_1^i - m_2^i)/(m_1^N - m_2^N),$$

where

$$\begin{aligned} m_1 &= \left(1 + \frac{\alpha h_c}{2\varepsilon} + \frac{\beta h_c^2}{4\varepsilon}\right) + \sqrt{\left(1 + \frac{\alpha h_c}{2\varepsilon} + \frac{\beta h_c^2}{4\varepsilon}\right)^2 + \left(-1 - \frac{\alpha h_c}{\varepsilon} + \frac{\beta h_c^2}{2\varepsilon}\right)} \\ &\geq 1 + \frac{\alpha h_c}{2\varepsilon} = \left(1 + \frac{\alpha(1-\tau)}{2\varepsilon N}\right) \geq \left(1 + \frac{\alpha\tau}{2\varepsilon N}\right), \\ m_2 &= \left(1 + \frac{\alpha h_c}{2\varepsilon} + \frac{\beta h_c^2}{4\varepsilon}\right) - \sqrt{\left(1 + \frac{\alpha h_c}{2\varepsilon} + \frac{\beta h_c^2}{4\varepsilon}\right)^2 + \left(-1 - \frac{\alpha h_c}{\varepsilon} + \frac{\beta h_c^2}{2\varepsilon}\right)}. \end{aligned}$$

Now

$$\mathbf{L}^N(\mathbf{Y}_r^{[k+2]} - \mathbf{Y}_r^{[k+1]})(x_i) = \mathbf{0}, \quad \forall\ x_i \in \bar{\Omega}_r^N, \quad (\mathbf{Y}_r^{[k+2]} - \mathbf{Y}_r^{[k+1]})(1) = \mathbf{0}.$$

Using our inductive hypothesis and (5.12)

$$\begin{aligned} |(\ \mathbf{Y}_r^{[k+2]} - \mathbf{Y}_r^{[k+1]})(1-2\tau)| &= |(\bar{\mathbf{Y}}_c^{[k+1]} - \bar{\mathbf{Y}}_c^{[k]})(1-2\tau)| \\ &= |(\mathbf{Y}_c^{[k+1]} - \mathbf{Y}_c^{[k]})(1-2\tau)| \leq \Phi_c^{[k+1]}(1-2\tau), \end{aligned}$$

where we have used the fact that $(1-2\tau)$ is the mesh point of $\bar{\Omega}_c^N$. Using Lemma 4 we obtain

$$\|\mathbf{Y}_r^{[k+2]} - \mathbf{Y}_r^{[k+1]}\|_{\bar{\Omega}_r^N} \quad \leq \quad \Phi_c^{[k+1]}(1-2\tau).$$

Here we used

$$\begin{aligned}\Phi_c^{[k+1]}(1-2\tau) &= \mathbf{C}\nu^k \frac{m_1^{N/2}-m_2^{N/2}}{m_1^N-m_2^N} \le \frac{\mathbf{C}\nu^k}{m_1^{N/2}} \\ &= \mathbf{C}\nu^k\Big(1+\frac{\tau\alpha}{2\varepsilon N}\Big)^{-N/2} = \mathbf{C}\nu^k\left(1+\frac{\tau\alpha}{2\varepsilon N}\right)^{-N/2} = \mathbf{C}\nu^{k+1}.\end{aligned}$$

Therefore we get

$$\|\mathbf{Y}_r^{[k+2]}-\mathbf{Y}_r^{[k+1]}\|_{\bar{\Omega}_r^N} \le C\nu^{k+1} \tag{5.13}$$

and consequently

$$\|\mathbf{Y}_r^{[k+2]}-\mathbf{Y}_r^{[k+1]}\|_{\bar{\Omega}_r^N\setminus\bar{\Omega}_c} \le C\nu^{k+1}. \tag{5.14}$$

Finally note that

$$\mathbf{L}^N(\mathbf{Y}_c^{[k+2]}-\mathbf{Y}_c^{[k+1]})(x_i)=\mathbf{0} \quad \text{for} \ \ x_i\in\Omega_c^N, \quad (\mathbf{Y}_c^{[k+2]}-\mathbf{Y}_c^{[k+1]})(0)=\mathbf{0}.$$

Using our inductive hypothesis and (5.13), we have

$$\begin{aligned}|(\mathbf{Y}_c^{[k+2]}-\mathbf{Y}_c^{[k+1]})(1-\tau)| &= |(\bar{\mathbf{Y}}_r^{[k+2]}-\bar{\mathbf{Y}}_r^{[k+1]})(1-\tau)| \\ &= |(\mathbf{Y}_r^{[k+2]}-\mathbf{Y}_r^{[k+1]})(1-\tau)| \le C\nu^{[k+1]},\end{aligned}$$

where we have used the fact that $(1-\tau)$ is the mesh point of $\bar{\Omega}_c^N$. Therefore, we can apply Lemma 4 to get

$$\|\mathbf{Y}_c^{[k+2]}-\mathbf{Y}_c^{[k+1]}\|_{\bar{\Omega}_c^N} \le C\nu^{k+1}. \tag{5.15}$$

Combining the estimates (5.14) and (5.15) we obtain,

$$\|\mathbf{Y}^{[k+2]}-\mathbf{Y}^{[k+1]}\|_{\bar{\Omega}^N} \le C\nu^{k+1}.$$

For $\tau=\frac{4\varepsilon}{\alpha}\ln N$ using the arguments given in Lemma 4.1 of [10] we obtain,

$$\nu=\Big(1+\frac{\tau\alpha}{2\varepsilon N}\Big)^{-N/2}=\left(1+\frac{2\ln N}{N}\right)^{-N/2} \le 2N^{-1}, \quad N\ge 1.$$

□

The following theorem is the main result of this paper, combining Lemmas 5 and 6 we prove that two iterations are sufficient to attain almost second-order convergence.

Theorem 2. *Let $\boldsymbol{y}(x)$ be the solution to (1.6)–(1.7) and $\mathbf{Y}^{[k]}(x_i)$ be the k^{th} iterate of the discrete Schwarz method described in Section 4. If $\tau=\frac{4\varepsilon}{\alpha}\ln N$ and $N>2$, then*

$$\|\,\mathbf{Y}^{[k]}-\boldsymbol{y}\|_{\bar{\Omega}^N} \le CN^{-k}+CN^{-2}\ln^3 N.$$

Proof. From Lemma 6 there exists $\mathbf{Y}$ such that $\mathbf{Y} := \mathrm{lt}_{k\to\infty}\mathbf{Y}^{[k]}$. We know from Lemma 5 that there exists C such that

$$\|\mathbf{Y}^{[k]} - \boldsymbol{y}\|_{\bar{\Omega}^N} \le C2^{-k} + CN^{-2}\ln^3 N.$$

This implies that

$$\|\mathbf{Y} - \boldsymbol{y}\|_{\bar{\Omega}^N} \le CN^{-2}\ln^3 N. \tag{5.16}$$

Also from Lemma 6 that there exists C such that

$$\|\mathbf{Y}^{[k+1]} - \mathbf{Y}^{[k]}\|_{\bar{\Omega}^N} \le CN^{-k}.$$

Consequently, for $N \ge 2$, there exists C such that

$$\|\mathbf{Y}^{[k]} - \mathbf{Y}\|_{\bar{\Omega}^N} \le C\sum_{l=k}^{\infty} N^{-l} = C\left[\frac{N^{-k}}{1-N^{-1}}\right] \le CN^{-k}. \tag{5.17}$$

Thus, using (5.16) and (5.17), we conclude that

$$\begin{aligned}\|\mathbf{Y}^{[k]} - \boldsymbol{y}\|_{\bar{\Omega}^N} &= \|\mathbf{Y}^{[k]} - \mathbf{Y} + \mathbf{Y} - \boldsymbol{y}\|_{\bar{\Omega}^N} \\ &\le \|\mathbf{Y}^{[k]} - \mathbf{Y}\|_{\bar{\Omega}^N} + \|\mathbf{Y} - \boldsymbol{y}\|_{\bar{\Omega}^N} \le CN^{-k} + CN^{-2}\ln^3 N.\end{aligned}$$

□

6 Numerical experiments

In this section, we consider one example to illustrate the theoretical results for the BVPs (1.1)–(1.2). The stopping criterion for the iterative procedure is taken to be

$$\|\mathbf{Y}^{[k+1]} - \mathbf{Y}^{[k]}\|_{\bar{\Omega}^N} \le 10^{-14}, \text{ for } \quad j = 1, 2.$$

We normally omit the superscript k on the final Schwarz iterate and write simply Y_j^N. Let Y_j^N be a Schwarz numerical approximation for the exact solution y_j on the mesh Ω^N and N is the number of mesh points. For a finite set of values of $\varepsilon = \{2^0, \ldots, 2^{-30}\}$, we compute the maximum point-wise two mesh difference errors for $j = 1, 2$

$$\|Y_j^N - y_j\|_{\Omega^N} \approx D_{\varepsilon,j}^N := \|Y_j^N - \bar{Y}_j^{2N}\|_{\Omega^N}, \quad D_j^N = \max_{\varepsilon} D_{\varepsilon,j}^N,$$

where $\bar{Y}_j^{2N}$ is the numerical solution obtained on a mesh with the same transition points, but with $2N$ intervals in each subdomain. From these quantities the ε-uniform order of convergence is computed from

$$p_j^N = \log_2\left\{D_j^N / D_j^{2N}\right\}, \text{ for } \quad j = 1, 2.$$

The computed maximum pointwise errors $D_j^N, (j = 1, 2)$ and the computed order of convergence $p_j^N, (j = 1, 2)$ and k (the number of iterations computed) for various values of N and ε are tabulated in Table 1 and Table 2. The nodal errors are plotted as graphs in Figure 1. We can see that the errors decrease as N increases. The computed rates of convergence are almost second-order, with the usual $\ln N$ factor associated with these techniques.

Example 1. Consider the BVP

$$-\varepsilon y^{iv}(x) + (2-x)y'''(x) + (1+x)y''(x) - (x^2/5)y(x) = -\sinh x,$$
$$y(0) = 0, \quad y''(0) = 0, \quad y(1) = 0, \quad y''(1) = 0.$$

The numerical results are presented in Table 1 and Table 2.

Table 1. Values of D_1^N, p_1^N for the solution component Y_1 for the Example 1

	Number of mesh points N				
	64	128	256	512	1024
2^0	4.5356e-008	1.1447e-008	2.8752e-009	7.2046e-010	1.8032e-010
2^{-2}	1.7836e-007	4.4157e-008	1.0975e-008	2.7349e-009	6.8258e-010
2^{-4}	2.3850e-007	5.8368e-008	1.4400e-008	3.5733e-009	8.8980e-010
2^{-6}	5.5922e-007	7.3269e-008	1.0758e-008	2.6651e-009	6.6270e-010
2^{-8}	3.6738e-006	8.3723e-007	1.9091e-007	4.3482e-008	9.8857e-009
2^{-10}	5.1796e-006	1.2608e-006	3.0798e-007	7.5364e-008	1.8455e-008
2^{-12}	5.6146e-006	1.3864e-006	3.4362e-007	8.5332e-008	2.1212e-008
2^{-14}	5.7272e-006	1.4191e-006	3.5296e-007	8.7960e-008	2.1942e-008
2^{-16}	5.7556e-006	1.4274e-006	3.5532e-007	8.8625e-008	2.2128e-008
2^{-18}	5.7627e-006	1.4294e-006	3.5591e-007	8.8792e-008	2.2174e-008
2^{-20}	5.7645e-006	1.4299e-006	3.5606e-007	8.8834e-008	2.2186e-008
2^{-22}	5.7650e-006	1.4301e-006	3.5610e-007	8.8845e-008	2.2189e-008
2^{-24}	5.7651e-006	1.4301e-006	3.5610e-007	8.8847e-008	2.2189e-008
2^{-26}	5.7651e-006	1.4301e-006	3.5611e-007	8.8848e-008	2.2190e-008
2^{-28}	5.7651e-006	1.4301e-006	3.5611e-007	8.8848e-008	2.2190e-008
2^{-30}	5.7651e-006	1.4301e-006	3.5611e-007	8.8848e-008	2.2190e-008
D_1^N	5.7651e-006	1.4301e-006	3.5611e-007	8.8848e-008	2.2190e-008
p_1^N	**2.0112**	**2.0057**	**2.0029**	**2.0014**	-

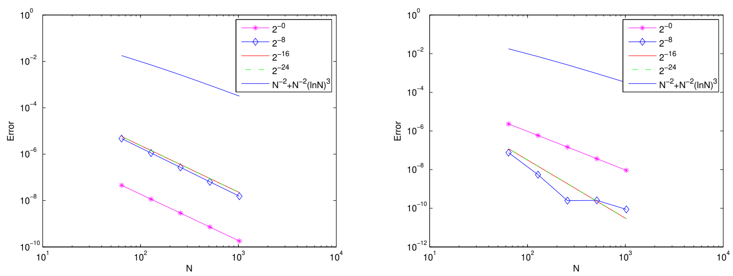

Figure 1. Nodal error for the components Y_1 and Y_2 of the Example 1

7 Conclusions

A singularly perturbed fourth-order ODEs of convection-diffusion problem is considered. It is shown that a designed discrete Schwarz method produces numerical approximations which converge in the maximum norm to the exact

Table 2. Values of D_2^N, p_2^N for the solution component Y_2 for the Example 1

	Number of mesh points N				
	64	128	256	512	1024
2^{0}	2.3114e-006	5.8302e-007	1.4641e-007	3.6683e-008	9.1808e-009
2^{-2}	6.1032e-006	1.5000e-006	3.7168e-007	9.2499e-008	2.3072e-008
2^{-4}	5.6156e-008	5.2490e-009	3.5293e-009	1.1476e-009	3.1933e-009
2^{-6}	6.5589e-008	1.9600e-008	5.0806e-009	1.2941e-009	3.2651e-010
2^{-8}	6.0556e-008	3.5318e-009	4.7276e-010	2.7546e-010	8.6772e-011
2^{-10}	1.0437e-007	1.1904e-008	1.2359e-009	9.5007e-011	2.5776e-012
2^{-12}	1.1620e-007	1.4187e-008	1.7060e-009	1.9786e-010	2.1020e-011
2^{-14}	1.1921e-007	1.4769e-008	1.8263e-009	2.2423e-010	2.7083e-011
2^{-16}	1.1997e-007	1.4916e-008	1.8565e-009	2.3086e-010	2.8610e-011
2^{-18}	1.2016e-007	1.4953e-008	1.8641e-009	2.3253e-010	2.8992e-011
2^{-20}	1.2021e-007	1.4962e-008	1.8660e-009	2.3294e-010	2.9088e-011
2^{-22}	1.2022e-007	1.4964e-008	1.8665e-009	2.3305e-010	2.9111e-011
2^{-24}	1.2022e-007	1.4965e-008	1.8666e-009	2.3307e-010	2.9117e-011
2^{-26}	1.2022e-007	1.4965e-008	1.8666e-009	2.3308e-010	2.9119e-011
2^{-28}	1.2022e-007	1.4965e-008	1.8666e-009	2.3308e-010	2.9119e-011
2^{-30}	1.2022e-007	1.4965e-008	1.8666e-009	2.3308e-010	2.9119e-011
D_2^N	6.1032e-006	1.5000e-006	3.7168e-007	9.2499e-008	2.3072e-008
p_2^N	**2.0246**	**2.0128**	**2.0066**	**2.0033**	-

solution. This convergence is shown to be of almost second-order. Note that from Theorem 2, for $k \geq 2$ the $N^{-2} + N^{-2} \ln^3 N$ term dominates the error bound. Thus, two iterations are sufficient to attained the desired accuracy.

The present method gives improved numerical results with regard to error and order compared with the other method in [2, 15, 16, 17]. From Theorem 2 it can be easily identified in which iterations, the Schwarz iterate terminates. From the given example number of iterations taken by this method is not more than two which is very much reduced when comparing iteration counts presented in [8, 9]. This illustrates the efficiency of the method used with proposed scheme in this paper.

Numerical experiment validate the theoretical result. The graphs plotted in the figure is convergent curves in the maximum norm at nodal points for the different values of ε and N for the example considered. This graph clearly indicate that the optimal error bound is of order $O(N^{-k} + N^{-2} \ln^3 N)$ as predicted.

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Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 6, 2829-2845
https://doi.org/10.28919/jmcs/4997
ISSN: 1927-5307

THE MULTIPLICATIVE SECOND HYPER ZAGREB INDEX OF SOME GRAPH OPERATIONS

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Abstract. In this paper, the multiplicative second hyper Zagreb index is presented and the sharp upper bound for this index of various graph operations for example, join, composition, cartesian and corona products of graphs are derived. And we prove that the sharp upper bound is tight.

Keywords: topological indices; graph operations.

2010 AMS Subject Classification: 05C07, 05C09, 05C76.

1. INTRODUCTION

All graphs observed here are simple, connected and finite. Let $V(G), E(G)$ and $d_G(w)$ indicate the vertex set, the edge set and the degree of a vertex of a graph G respectively. A graph with p vertices and q edges is known as a (p,q) graph. We encourage the readers to see[5] for basic definitions and notations of a graph.

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Received September 4, 2020

A topological index is a numerical parameter which is mathematically attained from the graph structure.

Gutman et.al.,[2] introduced the first and second Zagreb indices of a graph G as follows:

$$M_1(G) = \sum_{wz\in E(G)} (d_G(w)+d_G(z)) = \sum_{w\in V(G)} d_G^2(w) \text{ and } M_2(G) = \sum_{wz\in E(G)} d_G(w)d_G(z)$$

Shirdel et.al. in [7] found Hyper-Zagreb index $HM(G)$ which is established as

$$HM(G) = \sum_{wz\in E(G)} [d_G(w)+d_G(z)]^2.$$

Also, they have computed the hyper - Zagreb index of the cartesian product, composition, join and disjunction of graphs.

A forgotten topological index F-index [4] is defined for a graph G as

$$F(G) = \sum_{w\in V(G)} d_G^3(w) = \sum_{wz\in E(G)} [d_G^2(w)+d_G^2(z)]$$

Farahani et.al [3] defined the second hyper Zagerb as

$$HM_2(G) = \sum_{wz\in E(G)} [d_G(w)d_G(z)]^2.$$

Here we introduce a second forgotten topological index F_2 which is defined for a graph G as

$$F_2(G) = \sum_{w\in V(G)} d_G^4(w).$$

V.R.Kulli [6] introduced the first and second Gourava indices and defined as

$$GO_1(G) = \sum_{wz\in E(G)} (d_G(w)+d_G(z)) + (d_G(w)d_G(z))$$

and

$$GO_2(G) = \sum_{wz\in E(G)} d_G(w)d_G(z)(d_G(w)+d_G(z))$$

Todeschine et al [9,10] presented the multiplicative variants of ordinary Zagreb indices, which are defined as follows:

$$\prod_1 = \prod_1(G) = \prod_{w\in V(G)} d_G(w)^2 = \prod_{wz\in E(G)} \left[d_G(w)+d_G(z)\right]$$
$$\text{and } \prod_2 = \prod_2(G) = \prod_{wz\in E(G)} d_G(w)d_G(z)\big]$$

Recently, Akbar [1] has introduced the multiplicative hyper Zagreb index, denoted by

$$\prod HM(G) = \prod_{uv\in E(G)} (d_G(w)+d_G(z))^2$$

Also in this paper, the upper bounds on the multiplicative hyper Zagreb index of the cartesian, corona product, composition, join and disjunction of graphs.

In this paper, we introduce a new graph invariant namely multiplicative second hyper Zagreb index, denoted by

$$\prod HM_2(G) = \prod_{uv\in E(G)} (d_G(w)d_G(z))^2$$

In this paper, we compute the sharp upper bound for the multiplicative second hyper Zagreb index of the graph operations for example, join, composition, cartesian and corona products and prove that our bound is tight.

2. PRELIMINARIES

Lemma 2.1. [5, 8]

(a) $d_{G_1+G_2}(w) = \begin{cases} d_{G_1}(w)+V(G_2), & w\in V(G_2) \\ d_{G_2}(w)+V(G_1), & w\in V(G_2) \end{cases}$

(b) $d_{G_1[G_2]}(w,z) = V(G_2)d_{G_1}(w)+d_{G_2}(z)$

(c) $d_{G_1\Box G_2}((w_i,z_j)) = d_{G_1}(w_i)+d_{G_2}(z_j)$, *where* $(w_i,z_j)\in V(G_1\Box G_2)$.

(d)

$$d_{G_1\odot G_2}(w) = \begin{cases} d_{G_1}(w)+p_2 & if\ w\in V(G_1) \\ d_{G_1}(w)+p_2 & if\ w\in V(G_{2,i})\ for\ some\ 0\le i\le p_1-1, \end{cases}$$

where $u\in V(G_1\odot G_2)$ $G_{2,i}$ *is the ith copy of the graph* G_2 *in* $G_1\odot G_2$.

Lemma 2.2 (Arithmetic geometric Inequality)**.** *Let* $y_1,y_2,\ldots,y_n$ *be non-negative numbers. Then* $\dfrac{y_1+y_2+\cdots+y_n}{n} \ge \sqrt[n]{y_1y_2\cdots y_n}$

3. The Multiplicative Second Hyper Zagreb Index of Join of Graphs

Theorem 3.1. *Let $G_i, i = 1,2$ be a $(p_i, q_i)-$ graph. Then*

$$\prod HM_2(G_1+G_2) \leq \left[\frac{\begin{array}{r} HM_2(G_1)+p_2^2HM(G_1)+p_2^4q_1+2p_2GO_2(G_1) \\ +2p_2^2M_2(G_1)+2p_2^3M_1(G_1) \end{array}}{q_1}\right]^{q_1}$$
$$\times \left[\frac{\begin{array}{r} HM_2(G_2)+p_1^2HM(G_2)+p_1^4q_2 \\ +2p_1GO_2(G_2)+2p_1^2M_2(G_2)+2p_1^3M_1(G_2) \end{array}}{q_2}\right]^{q_2}$$
$$\times \left[\frac{\begin{array}{r} M_1(G_1)M_1(G_2)+p_1^2p_2M_1(G_1)+p_1p_2^2M_1(G_2) \\ +4p_1q_2M_1(G_1)+p_1^3p_2^3+p_2q_1M_1(G_2)+4p_1^2p_2^2q_2 \\ +4p_1^2p_2^2q_1+16p_1p_2q_1q_2 \end{array}}{p_1p_2}\right]^{p_1p_2}$$

Proof : From the definition of the second hyper Zagreb index,

$$\begin{aligned}
\prod HM_2(G_1+G_2) &= \prod_{wz\in E(G_1+G_2)} \left[d^2_{G_1+G_2}(w)\, d^2_{G_1+G_2}(z)\right] \\
&= \prod_{wz\in E(G_1)} \left[d^2_{G_1+G_2}(w) d^2_{G_1+G_2}(z)\right] \\
&\times \prod_{wz\in E(G_2)} \left[d^2_{G_1+G_2}(w)\, d^2_{G_1+G_2}(z)\right] \\
&\times \prod_{w\in V(G_1)} \prod_{z\in V(G_2)} \left[d^2_{G_1+G_2}(w)\, d^2_{G_1+G_2}(z)\right] \\
&= A\times B\times C
\end{aligned}$$

where A, B and C indicate the products of the above terms in order.

Now we calculate A.

$$\begin{aligned}
A &= \prod_{wz\in E(G_1)} \left[d^2_{G_1+G_2}(w)d^2_{G_1+G_2}(z)\right] \\
&= \prod_{wz\in E(G_1)} \left[(d_{G_1}(w)+p_2)^2(d_{G_1}(z)+p_2)^2\right] \\
&\le \left[\frac{\sum\limits_{wz\in E(G_1)} \left[(d_{G_1}(w)+p_2)^2(d_{G_1}(z)+p_2)^2\right]}{q_1}\right]^{q_1} \\
&= \left[\frac{\sum\limits_{wz\in E(G_1)} \left[d^2_{G_1}(w)+p_2^2+2p_2d_{G_1}(w)\right]\left[d^2_{G_1}(z)+p_2^2+2p_2d_{G_1}(z)\right]}{q_1}\right]^{q_1} \\
&= \left[\frac{\begin{array}{r} HM_2(G_1)+p_2^2HM(G_1)+p_2^4q_1+2p_2GO_2(G_1) \\ +2p_2^2M_2(G_1)+2p_2^3M_1(G_1)\end{array}}{q_1}\right]^{q_1}
\end{aligned}$$

Next we calculate B.

$$\begin{aligned}
B &= \prod_{wz\in E(G_2)} \left[d^2_{G_1+G_2}(w)d^2_{G_1+G_2}(z)\right] \\
&= \prod_{wz\in E(G_2)} \left[(d_{G_2}(w)+p_1)^2(d_{G_2}(z)+p_1)^2\right] \\
&\le \left[\frac{\sum\limits_{wz\in E(G_2)} \left[(d_{G_2}(w)+p_1)^2(d_{G_2}(z)+p_1)^2\right]}{q_2}\right]^{q_2} \\
&= \left[\frac{\sum\limits_{wz\in E(G_2)} \left[d^2_{G_2}(w)+p_1^2+2p_1d_{G_2}(w)\right]\left[d^2_{G_2}(z)+p_1^2+2p_1d_{G_2}(z)\right]}{q_2}\right]^{q_2}
\end{aligned}$$

$$= \left[\frac{\begin{array}{r} HM_2(G_2) + p_1^2 HM(G_2) + p_1^4 q_2 + 2p_1 GO_2(G_2) \\ +2p_1^2 M_2(G_2) + 2p_1^3 M_1(G_2) \end{array}}{q_2} \right]^{q_2}$$

Finally, we compute C.

$$\begin{aligned} C &= \prod_{w\in V(G_1)} \prod_{z\in V(G_2)} \left[d^2_{G_1+G_2}(w) + d^2_{G_1+G_2}(z)\right] \\ &= \prod_{w\in V(G_1)} \prod_{z\in V(G_2)} \left[(d_{G_1}(w)+p_2)^2 (d_{G_2}(z)+p_1)^2\right] \\ &\le \left[\frac{\sum\limits_{w\in V(G_1)} \sum\limits_{z\in V(G_2)} [(d_{G_1}(w)+p_2)^2 (d_{G_2}(z)+p_1)^2}{p_1 p_2} \right]^{p_1 p_2} \\ &= \left[\frac{\sum\limits_{w\in V(G_1)} \sum\limits_{z\in V(G_2)} \left[d^2_{G_1}(w) + p_2^2 + 2p_2 d_{G_1}(w)\right] [d^2_{G_2}(z) + p_1^2 + 2p_1 d_{G_2}(z)]}{p_1 p_2} \right]^{p_1 p_2} \\ &= \left[\frac{\begin{array}{r} M_1(G_1)M_1(G_2) + p_1^2 p_2 M_1(G_1) + p_1 p_2^2 M_1(G_2) + 4p_1 q_2 M_1(G_1) \\ +p_1^3 p_2^3 + p_2 q_1 M_1(G_2) + 4p_1^2 p_2^2 q_2 + 4p_1^2 p_2^2 q_1 + 16 p_1 p_2 q_1 q_2 \end{array}}{p_1 p_2} \right]^{p_1 p_2} \end{aligned}$$

Now using A, B and C we get the deired result. $\square$

Lemma 3.2. *Let $G_i, (i = 1,2)$ be two regular graphs of degree r_i.*
Let $G_i, (i = 1,2)$ be a $(p_i, q_i)-$ graph. Then

$$\begin{aligned} \prod HM_2(G_1+G_2) &= (r_1+p_2)^{4q_1} \times (r_2+p_1)^{4q_2} \\ &\quad \times [(r_1+p_2)^2 (r_2+p_1)^2]^{p_1 p_2} \end{aligned}$$

Proof :

$$\prod HM_2(G_1+G_2) = \prod_{wz\in E(G_1+G_2)} \left[d^2_{G_1+G_2}(w)\, d^2_{G_1+G_2}(z)\right]$$

$$= \prod_{wz\in E(G_1)} \left[d^2_{G_1+G_2}(w)d^2_{G_1+G_2}(z)\right]$$

$$\times \prod_{wz\in E(G_2)} \left[d^2_{G_1+G_2}(w)\,d^2_{G_1+G_2}(z)\right]$$

$$\times \prod_{w\in V(G_1)} \prod_{z\in V(G_2)} \left[d^2_{G_1+G_2}(w)\,d^2_{G_1+G_2}(z)\right]$$

$$= \prod_{wz\in E(G_1)} (r_1+p_2)^2(r_1+p_2)^2 \prod_{wz\in E(G_2)} (r_2+p_1)^2(r_2+p_1)^2$$

$$\prod_{wz\in E(G_1)} \prod_{wz\in E(G_2)} (r_1+p_2)^2(r_2+p_1)^2$$

$$= (r_1+p_2)^{4q_1} \times (r_2+p_1)^{4q_2}$$

$$\times [(r_1+p_2)^2(r_2+p_1)^2]^{p_1p_2} \tag{1}$$

□

Remark 3.3. We find the upper bound of Lemma 3.2 when G is a regular graph of degree r with p vertices and q edges. Here
$q = \frac{pr}{2}, M_1(G) = pr^2, M_2(G) = qr^2, F(G) = 2qr^2, F_2(G) = 2qr^3,$
$HM(G) = 4qr^2 HM_2(G) = qr^4, GO_2(G) = 2qr^3$

Corollary 3.4. *Let $G_i,(i=1,2)$ be two regular graphs of degree r_i.*
Let $G_i,(i=1,2)$ be a $(p_i,q_i)-$ graph. Then

$$\prod HM_2(G_1+G_2) \leq (r_1+p_2)^{4q_1} \times (r_2+p_1)^{4q_2}$$

$$\times [(r_1+p_2)^2(r_2+p_1)^2]^{p_1p_2} \tag{2}$$

From (1) and (2) the bound is tight.

4. The Multiplicative Second Hyper Zagreb Index of Composition of Graphs

Theorem 4.1. *Let $G_i, i = 1,2$ be a $(p_i, q_i)-$ graph. Then*

$$\prod HM_2(G_1[G_2]) \leq \left[\frac{\begin{array}{l} p_2^4 q_2 F_2(G_1) + p_2^2 M_1(G_1) HM(G_2) + p_1 HM_2(G_2) \\ +2p_2^3 F(G_1) M_1(G_2) + 2p_2^2 M_1(G_1) M_2(G_2) + 4p_2 q_1 GO_2(G_2) \end{array}}{p_1 q_2}\right]^{p_1 q_2}$$

$$\times \left[\frac{\begin{array}{l} p_2^6 HM_2(G_1) + p_2^3 M_1(G_2) F(G_1) + q_1 (M_1(G_2))^2 + 4p_2^4 q_2 GO_2(G_2) \\ +16 p_2^2 q_2^2 M_2(G_1) + 4p_2 q_2 M_1(G_1) M_1(G_2) \end{array}}{q_1 p_2^2}\right]^{q_1 p_2^2}$$

Proof :

$$\begin{aligned}
\prod HM_2(G_1[G_2]) &= \prod_{(w,k)(z,l)\in E(G_1[G_2])} \left[d^2_{G_1[G_2]}(w,k)\, d^2_{G_1[G_2]}(z,l)\right] \\
&= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left[d^2_{G_1[G_2]}(w,k) d^2_{G_1[G_2]}(z,l)\right] \\
&\times \prod_{k\in V(G_2)} \prod_{l\in V(G_2)} \prod_{wz\in E(G_1)} [d^2_{G_1[G_2]}(w,k) d^2_{G_1[G_2]}(z,l)] \\
&= A \times B,
\end{aligned}$$

where A and B indicate the products of the above terms in order.

Now we compute A.

$$\begin{aligned}
A &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left[d^2_{G_1[G_1]}(w,k)\, d^2_{G_1[G_2]}(w,l)\right] \\
&= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} [[p_2 d_{G_1}(w) + d_{G_2}(k)]^2 [p_2 d_{G_1}(w) + d_{G_2}(l)]^2] \\
&\leq \left[\frac{\sum_{w\in V(G_1)} \sum_{kl\in E(G_2)} \left[[p_2 d_{G_1}(w) + d_{G_2}(k)]^2 [p_2 d_{G_1}(w) + d_{G_2}(k)]^2\right]}{p_1 q_2}\right]^{p_1 q_2}
\end{aligned}$$

$$= \left[\frac{\sum\limits_{w\in V(G_1)} \sum\limits_{kl\in E(G_2)} \left[p_2^2 d_{G_1}^2(w) + d_{G_2}^2(k) + 2p_2 d_{G_1}(w) d_{G_2}(k)\right] \left[p_2^2 d_{G_1}^2(w) + d_{G_2}^2(l) + 2p_2 d_{G_1}(w) d_{G_2}(l)\right]}{p_1 q_2} \right]^{p_1 q_2}$$

$$= \left[\frac{p_2^4 q_2 F_2(G_1) + p_2^2 M_1(G_1) HM(G_2) + p_1 HM_2(G_2) + 2p_2^3 F(G_1) M_1(G_2) + 2p_2^2 M_1(G_1) M_2(G_2) + 4p_2 q_1 GO_2(G_2)}{p_1 q_2} \right]^{p_1 q_2}$$

$$B = \prod_{k\in V(G_2)} \prod_{l\in V(G_2)} \prod_{wz\in E(G_1)} \left[d_{G_1[G_1]}^2(w,k)\, d_{G_1[G_2]}^2(z,l)\right]$$

$$= \prod_{k\in V(G_2)} \prod_{l\in V(G_2)} \prod_{wz\in E(G_1)} \left[[p_2 d_{G_1}(w) + d_{G_2}(k)]^2 [p_2 d_{G_1}(z) + d_{G_2}(l)]^2\right]$$

$$\leq \left[\frac{\sum\limits_{k\in V(G_2)} \sum\limits_{l\in V(G_2)} \sum\limits_{wz\in E(G_1)} \left[[p_2 d_{G_1}(w) + d_{G_2}(k)]^2 [p_2 d_{G_1}(z) + d_{G_2}(l)]^2\right]}{p_2^2 q_1} \right]^{p_2^2 q_1}$$

$$= \left[\frac{\sum\limits_{k\in V(G_2)} \sum\limits_{l\in V(G_2)} \sum\limits_{wz\in E(G_1)} \left[p_2^2 d_{G_1}^2(p) + d_{G_2}^2(k) + 2p_2 d_{G_1}(w) d_{G_2}(k)\right] \left[p_2^2 d_{G_1}^2(z) + d_{G_2}^2(l) + 2n_2 d_{G_1}(z) d_{G_2}(l)\right]}{q_1 p_2^2} \right]^{q_1 p_2^2}$$

$$= \left[\frac{p_2^6 HM_2(G_1) + p_2^3 M_1(G_2) F(G_1) + q_1 (M_1(G_2))^2 + 4p_2^4 q_2 GO_2(G_2) + 16 p_2^2 q_2^2 M_2(G_1) + 4p_2 q_2 M_1(G_1) M_1(G_2)}{q_1 p_2^2} \right]^{q_1 p_2^2}$$

Using A and B, we get the required result. □

Lemma 4.2. *Let $G_i, i = 1,2$ be two regular graphs of degree r_i and let $G_i, i = 1,2$ be a (p_i, q_i)-graph. Then $\prod HM_2(G_1[G_2]) = (p_2r_1 + r_2)^{4(p_1q_2 + p_2^2q_1)}$.*

Proof :

$$
\begin{aligned}
\prod HM_2(G_1[G_2]) &= \prod_{w \in V(G_1)} \prod_{kl \in E(G_2)} \left[d^2_{G_1[G_2]}(w,k) d^2_{G_1,[G_2]}(w,l)\right] \\
&\quad \times \prod_{k \in V(G_2)} \prod_{l \in V(G_2)} \prod_{wz \in E(G_1)} \left[d^2_{G_1[G_2]}(w,k) d^2_{G_1[G_2]}(z,l)\right] \\
&= \prod_{w \in V(G_1)} \prod_{kl \in E(G_2)} (p_2r_1 + r_2)^2 (p_2r_1 + r_2)^2 \\
&\quad \times \prod_{k \in V(G_2)} \prod_{l \in V(G_2)} \prod_{wz \in E(G_1)} (p_2r_1 + r_2)^2 (p_2r_1 + r_2)^2 \\
&= (p_2r_1 + r_2)^{4p_1q_2} \times (p_2r_1 + r_2)^{4p_2^2q_1} \\
&= (p_2r_1 + r_2)^{4(p_1q_2 + p_2^2q_1)} \qquad (3)
\end{aligned}
$$

□

Corollary 4.3. *Let $G_i, (i = 1,2)$ be two regular graphs of degree r_i. Let $G_i, (i = 1,2)$ be a $(p_i, q_i)-$ graph. Then*

$$\prod HM_2(G_1[G_2]) \leq (p_2r_1 + r_2)^{4(p_1q_2 + p_2^2q_1)} \qquad (4)$$

From (3) and (4) our bound is tight.

5. The Multiplicative Second Hyper Zagreb Index of Cartesian Product of Graphs

Theorem 5.1. *Let $G_i, i = 1,2$ be a $(p_i, q_i)-$graph. Then*

$$\prod HM_2(G_1 \Box G_2) \leq \left[\frac{\begin{array}{c} q_2F_2(G_1) + 2F(G_1)M_1(G_2) + 4M_1(G_1)M_2(G_2) \\ +M_1(G_1)F(G_2) + p_1HM_2(G_2) + 4q_1GO_2(G_2) \end{array}}{p_1q_2}\right]^{p_1q_2}$$

$$\times \left[\frac{\begin{array}{c} p_1F_2(G_2)+2F(G_2)M_1(G_1)+4M_1(G_2)M_2(G_1) \\ +M_1(G_2)F(G_1)+p_2HM_2(G_1)+4q_2GO_2(G_1) \end{array}}{p_2q_1} \right]^{p_2q_1}$$

Proof :

$$\begin{aligned}
\prod HM_2(G_1\Box G_2) &\leq \prod_{(w,k)(z,l)\in E(G_1\Box G_2)} \left[d^2_{G_1\Box G_2}(w,k)d^2_{G_1\Box G_2}(z,l)\right] \\
&= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left[d^2_{G_1\Box G_2}(w,k)d^2_{G_1\Box G_2}(z,l)\right] \\
&\times \prod_{k\in V(G_2)} \prod_{wz\in E(G_1)} \left[d^2_{G_1\Box G_2}(w,k)d^2_{G_1\Box G_2}(z,l)\right] \\
&= A\times B
\end{aligned}$$

where A and B indicate the products of the above terms in order.

Now we calculate A.

$$\begin{aligned}
A &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left[d^2_{G_1\Box G_2}(w,k)d^2_{G_1\Box G_2}(z,l)\right] \\
&= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left[[d_{G_1}(w)+d_{G_2}(k)]^2[d_{G_1}(w)+d_{G_2}(l)]^2\right] \\
&\leq \left[\frac{\sum\limits_{w\in V(G_1)} \sum\limits_{kl\in E(G_2)} [d_{G_1}(w)+d_{G_2}(k)]^2[d_{G_1}(w)+d_{G_2}(l)]^2}{p_1q_2} \right]^{p_1q_2} \\
&= \left[\frac{\begin{array}{r} \sum\limits_{w\in V(G_1)} \sum\limits_{kl\in E(G_2)} \left[d^2_{G_1}(w)+d^2_{G_2}(k)+2d_{G_1}(w)d_{G_2}(k)\right] \\ \left[d^2_{G_1}(k)+d^2_{G_2}(l)+2d_{G_1}(w)d_{G_2}(l)\right] \end{array}}{p_1q_2} \right]^{p_1q_2}
\end{aligned}$$

$$= \left[\frac{\begin{array}{l} q_2F_2(G_1)+2F(G_1)M_1(G_2)+4M_1(G_1)M_2(G_2)+M_1(G_1)F(G_2) \\ +p_1HM_2(G_2)+4q_1GO_2(G_2) \end{array}}{p_1q_2}\right]^{p_1q_2}$$

Now we compute B.

$$\begin{aligned}
B &= \prod_{k\in V(G_2)}\prod_{wz\in E(G_1)} \left[d^2_{G_1\square G_2}(w,k)d^2_{G_1\square G_2}(z,l)\right] \\
&= \prod_{k\in V(G_2)}\prod_{wz\in E(G_1)} \left[[d_{G_1}(w)+d_{G_2}(k)]^2[d_{G_1}(z)+d_{G_2}(k)]^2\right] \\
&\leq \left[\frac{\sum\limits_{k\in V(G_2)}\sum\limits_{wz\in E(G_1)} [d_{G_1}(w)+d_{G_2}(k)]^2[d_{G_1}(z)+d_{G_2}(k)]^2}{p_2q_1}\right]^{p_2q_1} \\
&= \left[\frac{\begin{array}{r} \sum\limits_{k\in V(G_2)}\sum\limits_{wz\in E(G_1)} \left[d^2_{G_1}(w)+d^2_{G_2}(k)+2d_{G_1}(w)d_{G_2}(k)\right] \\ \left[d^2_{G_1}(z)+d^2_{G_2}(k)+2d_{G_1}(z)d_{G_2}(k)\right] \end{array}}{p_2q_1}\right]^{p_2q_1} \\
&= \left[\frac{\begin{array}{l} q_1F_2(G_2)+2F(G_2)M_1(G_1)+4M_1(G_2)M_2(G_1)+M_1(G_2)F(G_1) \\ +p_2HM_2(G_1)+4q_2GO_2(G_1) \end{array}}{p_2q_1}\right]^{p_2q_1}
\end{aligned}$$

Using A and B we get the desired result. □

Lemma 5.2. *Let $G_{i_1} i = 1,2$ be two regular graphs of degree r_i and let $G_i; = 1,2$ be a $(p_i,q_i)-$ graph. Then $\prod HM_2(G_1\square G_2) = (r_1+r_2)^{4(p_1q_2+p_2q_1)}$*

Proof :

$$\begin{aligned}
\prod HM_2(G_1\square G_2) &= \prod_{w\in V(G_1)}\prod_{kl\in E(G_2)} \left[d^2_{G_1\square G_2}(w,k)\,d^2_{G_1\square G_2}(w,l)\right] \\
&\times \prod_{k\in V(G_2)}\prod_{wz\in E(G_1)} [d^2_{G_1\square G2}(w,k)+d_{G_1^2\square G_2}(z,k)]
\end{aligned}$$

$$= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} (r_1+r_2)^2 (r_1+r_2)^2$$

$$\times \prod_{k\in V(G_2)} \prod_{wz\in E(G_2)} (r_1+r_2)^2 (r_1+r_2)^2$$

$$= (r_1+r_2)^{4p_1q_2} \times (r_1+r_2)^{4p_2q_1}$$

$$= (r_1+r_2)^{4(p_1q_2+p_2q_1)} \tag{5}$$

□

Corollary 5.3. *Let $G_i, (i=1,2)$ be two regular graphs of degree r_i.*
Let $G_i, (i=1,2)$ be a $(p_i,q_i)-$ graph. Then

$$\prod HM_2(G_1 \Box G_2) \leq (r_1+r_2)^{4(p_1q_2+p_2q_1)} \tag{6}$$

From (5) and (6) the bound is tight.

6. THE MULTIPLICATIVE SECOND HYPER ZAGREB INDEX OF CORONA PRODUCT OF GRAPHS

Theorem 6.1. *Let $G_i, i=1,2$ be a (p_i,q_i)-graph. Then*

$$\prod HM_2(G_1 \odot G_2) \leq \left[\frac{\begin{array}{r} p_2^4q_1 + 2p_2^3M_1(G_1) + 4p_2^2M_2(G_1) + p_2^2F(G_1) \\ +HM_2(G_1) + 2p_2GO_2(G_1) \end{array}}{q_1} \right]^{q_1}$$

$$\times \left[\frac{\begin{array}{r} q_2 + 2M_1(G_2) + 4M_2(G_2) + F(G_2) \\ +HM_2(G_2) + 2GO_2(G_2) \end{array}}{q_2} \right]^{p_1q_2}$$

$$\left[\frac{\begin{array}{r}M_1(G_1)M_1(G_2)+4q_2M_1(G_2)+p_2M_1(G_1)+p_2M_1(G_2)(p_1p_2+4q_1)\\ +16q_1q_2p_2+4p_2^2q_1+p_1p_2^2(p_2+4q_2)\end{array}}{p_1p_2}\right]^{p_1p_2}$$

Proof :

$$\begin{aligned}\prod HM_2(G_1\odot G_2) &= \prod_{wz\in E(G_1)}\big((d_{G_1}(w)+p_2)^2(d_{G_1}(z)+p_2)^2\big)\\ &\times \prod_{w\in V(G_1)}\prod_{kl\in E(G_2)}\big((d_{G_2}(k)+1)^2(d_{G_2}(l)+1)^2\big)\\ &\times \prod_{w\in V(G_1)}\prod_{k\in V(G_2)}\big((d_{G_1}(w)+p_2)^2(d_{G_2}(k)+1)^2\big)\\ &= A\times B\times C\end{aligned}$$

where A,B and C are the products of the about terms in order.

Now calculate A,

$$\begin{aligned}A &= \prod_{wz\in E(G_1)}\big((d_{G_1}(w)+p_2)^2(d_{G_1}(z)+p_2)^2\big)\\ &\leq \left[\frac{\sum_{wz\in E(G_1)}(d_{G_1}(w)+p_2)^2(d_{G_1}(z)+p_2)^2}{q_1}\right]^{q_1}\\ &= \left[\frac{\sum_{wz\in E(G_1)}[d_{G_1}^2(u)+p_2^2+2p_2d_{G_1}(w)][d_{G_1}^2(z)+p_2^2+2p_2d_{G_1}(z)]}{q_1}\right]^{q_1}\\ &= \left[\frac{p_2^4q_1+2p_2^3M_1(G_1)+4p_2^2M_2(G_1)+p_2^2F(G_1)+HM_2(G_1)+2p_2GO_2(G_1)}{q_1}\right]^{q_1}\end{aligned}$$

Next compute B.

$$\begin{aligned}B &= \prod_{w\in V(G_1)}\prod_{kl\in E(G_2)}\big((d_{G_2}(k)+1)^2(d_{G_2}(l)+1)^2\big)\\ &\leq \left[\frac{\sum_{w\in V(G_1)}\sum_{kl\in E(G_2)}(d_{G_2}(k)+1)^2(d_{G_2}(l)+1)^2}{p_1q_2}\right]^{p_1q_2}\end{aligned}$$

$$= \left[\frac{\sum_{w\in V(G_1)}\sum_{kl\in E(G_2)}\Big[1+2(d_{G_2}(k)+d_{G_2}(l))+4d_{G_2}(k)d_{G_2}(l) \\ (d^2_{G_2}(k)+d^2_{G_2}(l))+d^2_{G_2}(k)d^2_{G_2}(l) \\ +2(d^2_{G_2}(k)d_{G_2}(l)+d_{G_2}(k)d^2_{G_2}(l))\Big]}{p_1q_2}\right]^{p_1q_2}$$

$$= \left[\frac{q_2+2M_1(G_2)+4M_2(G_2)+F(G_2)+HM_2(G_2)+2GO_2(G_2)}{q_2}\right]^{p_1q_2}$$

Finally, compute C

$$C = \prod_{w\in V(G_1)}\prod_{k\in V(G_2)}\big(d_{G_1}(w)+p_2)^2(d_{G_2}(k)+1)^2\big)$$

$$\leq \left[\frac{\sum_{w\in V(G_1)}\sum_{k\in V(G_2)}\big(d_{G_1}(w)+p_2)^2(d_{G_2}(k)+1)^2\big)}{p_1p_2}\right]^{p_1p_2}$$

$$= \left[\frac{\sum_{w\in V(G_1)}\sum_{k\in V(G_2)}\Big(d^2_{G_1}(w)+2p_2d_{G_1}(w)+p_2^2)(d^2_{G_2}(k)+2d_{G_2}(k)+1)\Big)}{p_1p_2}\right]^{p_1p_2}$$

$$= \left[\frac{M_1(G_1)M_1(G_2)+4q_2M_1(G_2)+p_2M_1(G_1)+p_2M_1(G_2)(p_1p_2+4q_1) \\ +16q_1q_2p_2+4p_2^2q_1+p_1p_2^2(p_2+4q_2)}{p_1p_2}\right]^{p_1p_2}$$

Now multiplying A,B and C we get the required result. □

Lemma 6.2. *Let $G_i, i=1,2$ be two regular graph of degree r_i, and let $G_i, i=1,2$ be a $(p_i,q_i)-$ graph. Then*

$$\prod HM_2(G_1\odot G_2) = (r_1+p_2)^{4q_1}\times(r_2+1)^{4p_1q_2}\times((r_1+p_2)^2(r_2+1)^2)^{p_1p_2}$$

Proof :

$$\prod HM_2(G_1\odot G_2) = \prod_{wz\in E(G_1)}\big((d_{G_1}(w)+p_2)^2(d_{G_1}(z)+p_2)^2\big)$$

$$\times \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left((d_{G_2}(k)+1)^2(d_{G_2}(l)+1)^2\right)$$

$$\times \prod_{w\in V(G_1)} \prod_{k\in V(G_2)} \left((d_{G_1}(w)+p_2)^2(d_{G_2}(k)+1)^2\right)$$

$$= \prod_{wz\in E(G_1)} (r_1+p_2)^2(r_1+p_2)^2$$

$$\times \prod_{uwinV(G_1)} \prod_{kl\in E(G_2)} (r_2+1)^2(r_2+1)^2$$

$$\times \prod_{w\in V(G_1)} \prod_{k\in V(G_2)} (r_1+p_2)^2(r_2+1)^2$$

$$= (r_1+p_2)^{4q_1} \times (r_2+1)^{4p_1q_2} \times ((r_1+p_2)^2(r_2+1)^2)^{p_1p_2} \tag{7}$$

$\square$

Corollary 6.3. *Let $G_i, (i=1,2)$ be two regular graphs of degree r_i.*
Let $G_i, (i=1,2)$ be a $(p_i,q_i)-$ graph. Then

$$\prod HM_2(G_1 \odot G_2) \leq (r_1+p_2)^{4q_1} \times (r_2+1)^{4p_1q_2} \times ((r_1+p_2)^2(r_2+1)^2)^{p_1p_2} \tag{8}$$

From (7) and (8) the bound is tight.

7. Conclusion

In this paper,we have defined the multiplicative second hyper Zagreb index and derived the sharp upper bound for this index of various graph operations lke join, composition, cartesian and corona producst of graphs are derived. And we have proved that the sharp upper bound is tight.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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Quantitative Microscopy: Particle

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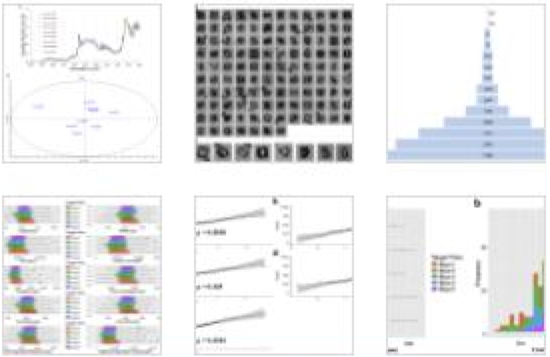

ELSEVIER

Journal of Pharmaceutical Sciences
Volume 110, Issue 2, February 2021, Pages 833-849

Research Article
Pharmaceutics, Drug Delivery and Pharmaceutical Technology

Quantitative Microscopy: Particle Size/Shape Characterization, Addressing Common Errors Using 'Analytics Continuum' Approach

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https://doi.org/10.1016/j.xphs.2020.09.022

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Abstract

Particle size/shape characterization of active pharmaceutical ingredient (API) is integral to successful product development. It is more of a correlative property than a decision-making measure. Though microscopy is the only technique that

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than a decision-making measure. Though microscopy is the only technique that provides a direct measure of particle properties, it is neglected for reasons like non-repeatability and non-reproducibility which is often attributed to a) fundamental error, b) segregation error, c) human error, d) sample randomness, e) sample representativeness etc. Using the "Sucrose" as model sample, we propose "analytics continuum" approach that integrates optical microscope PSD measurements complimented by NIR spectroscopy-based trending analysis as a prescreening tool to demonstrate sample randomness and representativeness. Furthermore, plethora of statistical tests are utilized to infer population statistics. Subsequently, an attribute-based control chart and bootstrap-based confidence interval was developed to monitor product performance. A flowchart to serve as an elementary guideline is developed, which is then extended to handle more complex situations involving API crystallized from two different solvent systems. The results show that the developed methodology can be utilized as a quantitative procedure to assess the suitability of API/excipients from different batches or from alternate vendors and can significantly help in understanding the differences between material even on a minor scale.

Previous article in issue
Next article in issue

Keywords

Particle size; Image analysis; Morphology; Near-infrared spectroscopy (NIRS); Principal component analysis; Sucrose

Pregelatinized Starch: Variability in Gelatinization and its Influence on Product Performance

Article *in* International Journal of Research in Pharmaceutical Sciences · January 2021

DOI: 10.26452/ijrps.v12i1.3969

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ORIGINAL ARTICLE

INTERNATIONAL JOURNAL OF RESEARCH IN PHARMACEUTICAL SCIENCES

Published by JK Welfare & Pharmascope Foundation Journal Home Page: www.pharmascope.org/ijrps

Pregelatinized Starch: Variability in Gelatinization and its Influence on Product Performance

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Article History:

Received on: 03 Aug 2020
Revised on: 01 Sep 2020
Accepted on: 11 Sep 2020

Keywords:

Functional Characteristics, Starch Retrogradation, Crystallinity, Particle Size, Birefringence, Gelatinization, Disintegrant-Binder properties, Viscosity, Disintegration test

ABSTRACT

Starch and its derivatives are one of the significant excipients used in the pharmaceutical formulations due to their multi-purpose functionalities. The purpose of this study is two-fold: (1) Firstly, to propose a systematic approach in understanding the material properties of a starch derivative (pregelatinised starch/PGS) using analytical 'toolbox' as part of 'alternative supplier sourcing', and secondly (2) To demonstrate the effect of PGS from different vendors on the tablet disintegration using model formulations. Contextually, a two-tier characterisation procedure is generally considered as a prerequisite for establishing the sameness of the material obtained either from different batches or from various vendors. Primarily, the sameness between typical quality-control tests and compendial requirements are to be established. If similar, then sameness between the functional characteristics is to be established. In this context, the PGS from two vendors met the specifications, and there were no differences for the test results in the certificates of analysis. However, when subjected to functionality assessment, the two lots were found to be distinctly different. The influence of the functional property variations was further exemplified from viscosity results of raw material. Furthermore, this difference was even more evident when the model formulations were subjected to disintegration testing. The similarity in compendial tests but significant differences in functionality characteristics for the PGS of two vendors can be unravelled by considering variations in particle size, crystallinity, starch retrogradation and these changes are potentially attributed to the differences in the gelatinisation procedures adopted by the vendors.

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ISSN: 0975-7538

DOI: https://doi.org/10.26452/ijrps.v12i1.3969

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INTRODUCTION

Alternate supplier evaluation for active pharmaceutical ingredients (APIs) and excipients is one of the critical projects that are currently sponsored within many pharmaceutical companies (Saravanan *et al.*, 2019). The motivation behind "Alternate Supplier Sourcing Projects or simply Alternate Sourcing" are numerous, few of which are: cost savings, the addition of more than one supplier, streamlining or reduction in the number of suppliers, overcoming supplier monopoly, quality issues with current suppliers, business contingency plans etc. (Moreton, 2019). "Alternate sourcing" are activities wherein new product development is not involved, however, involves the replacement of one or more components of an existing formulation that can be either API or excipients. APIs are the ingredients that are intended to furnish pharmacological actions. Whereas, excipients are added to pharmaceutical drug formulations to assist in the manufacturing, improve dosage form's stability, bioavailability, patient acceptability etc. (Kubbinga *et al.*, 2014). Excipients have been historically characterised as non-functional constituents that have no impact on the therapeutic activity of the medicine, and, in this context, the substitutions are carried over (Kushner, 2013). The primary assumption behind such projects is "excipients sameness" based on "Certificate of Analysis or CoA" (Bejarano *et al.*, 2019).

However, time and again, there has been a body of literature evidence indicating excipient functional characteristics playing a critical role on manufacturability, processability and product performance (Ruban *et al.*, 2018). There is now an increased awareness about the role of excipients and their influence they exert on the developed formulations (Abrantes *et al.*, 2016). Although the supplier change does not have a regulatory impact, it does require experimental validation (Charoo, 2020). The route to identity, purity, etc. are addressed and are provided as 'excipient specifications' in the pharmacopoeias which are also provided as CoA by the vendors (Ramesh *et al.*, 2019). However, assessment for excipient function or functionality specifications are not provided (Elder *et al.*, 2016). One reason could be that since the excipients play a multitude of roles in the formulation, hence, it is controversial to include the functional property assessment in the monograph (Gamble *et al.*, 2010). Nevertheless, it is the necessity of the formulator to understand the functional property requirement as well as its assessment to provide products with consistent quality. To overcome such undesirable events, guideline documents (2015/C 95/02) have been formalised to regulate supply chain of excipients with the idea of ascertaining risk assessment procedures, those, similar to APIs (Kader, 2016).

In most cases, the mono source accessibility of critical raw material is regarded as a significant financial and quality risk (Jaberidoost *et al.*, 2013). As part of business contingency plans in meeting market demands, a common strategy widely adopted is "pharmaceutical alternate sourcing" and to have more than one suppliers/vendor (Wöhl-Bruhn *et al.*, 2013). That said, to provide a successful exchange of one excipient with others in the formulation, it is often required for suppliers to markdown the source of origin of excipients. Historically, excipients were obtained from minerals, plants, microorganisms, chemical modification of a natural compound upon purification, or purely chemical synthesis etc., the final product variability either pre-exists (natural sourcing) or is created during manufacturing or may arise due to inadequate excipient functional specifications. These variabilities can exist between suppliers (inter-variability) or within suppliers' batch (intra-variability) (Zhao and Augsburger, 2006). As per the law of variation, everything varies as well as it is random; subsequently, control of variability becomes difficult. It is imperative to delineate the variability, causing compromise on product quality from white noise.

As per ICH Q8 requirements (Srinivasan and Iser, 2009), the quality compromising variability should be described as critical. In such scenarios, the following attributes are identified as part of the requirements which are (i) critical material attributes (referring to quality compromised due to input materials, CMAs), (ii) critical process parameters (referring to quality compromised due to manufacturing process, CPPs), (iii) critical quality attributes (referring to quality compromised due to output product, CQAs) (Simões *et al.*, 2020). To consistently deliver on the intended product performance (CQAs) as well as avoiding batch failures and product recalls, understanding the impact of CMAs, as well as CPPs, are required. The correlation of manufacturing process on final product characteristics is well-understood and well-documented. Interestingly, while developing a robust formulation, a thorough consideration of the variabilities associated with both raw material and process should be performed as well as documented. In doing so, either of the two common approaches is utilised (i) traditional approach: involves tightening the specifications concerning API, excipients and process, (ii) QbD approach: expected variabilities from API as well as excipients are incorporated and; appropriate process end-point identified (Simões *et al.*, 2020). The advantage concerning QbD approach is that, during manufacturing, the complex interplay between API, excipients, and process are controlled effectively, while, the traditional approach might not address the interactions (Yu *et al.*, 2014).

Interestingly, FDA adopted the question-based review (QbR) which requires the sponsors to understand, integrate and implement the effect of raw material variability to assure product quality through optimal process design and performance-based specifications (Srinivasan and Iser, 2009). For this, a detailed understanding of a substance's chemical, structural, molecular, particulate, mechanical, and bulk properties are to be explored and, will be a prerequisite to link the material properties to CPPs and CQAs (Simões *et al.*, 2020). Various researchers have extensively characterised the impact of the excipient material property variations, lot-to-lot/batch-to-batch variations, source-to-source variations for microcrystalline cellulose (MCC) (Haware *et al.*, 2010), native and pregelatinised starch (Adedokun and Itiola, 2010), lactose (Ticehurst *et al.*, 1996), sodium starch glycollate (SSG) (Shah and Augsburger, 2002), magnesium stearate (Zarmpi *et al.*, 2020), dicalcium phosphate (DCP) (Landín *et al.*, 1994), hydroxy propyl cellulose (HPC) (Alvarez-Lorenzo *et al.*, 1998), carbomer (Pérez-Marcos *et al.*, 1993), glyceryl monostearate (O'laughlin *et al.*, 1989), polyethene glycol (PEG) (Wöhl-Bruhn *et al.*, 2013), croscarmellose sodium (Zhao and Augsburger, 2006), Xanthum gum (Thacker *et al.*, 2010), crospovidone (Shah and Augsburger, 2001) etc., using various analytical methods in a way to understand as well as control the variability on the performance and manufacturability of dosage forms.

The primary objective of this work is to have a robust yet straightforward 'toolbox' to identify the critical material attributes of pregelatinised starch (PGS) and its suitability to screen the incoming raw materials. There are various types of starch like waxy starch, high amylose containing, and high amylopectin containing starches. These variations can have a profound influence on product performance; that is, the high amylose starch is used as a binder while high amylopectin provides disintegrating properties. Similarly, the pregelatinised starch can undergo retrogradation which could potentially influence the properties. Analytical 'toolbox' employed to characterise PGS were X-Ray Powder Diffraction (XRPD), Raman spectroscopy, polarised light microscopy, laser diffraction, thermogravimetry, viscosity measurements etc. A comprehensive understanding of the functionality of pregelatinised starch (PGS) would be essential. Based on the understanding obtained from the solid state studies, a model API, and excipients along with PGS from two vendors (labelled as Vendor 1 and Vendor 2) were studied. Once the functional assessment was carried out, model formulations were prepared and characterised.

MATERIALS AND METHODS

Materials

Two different batches were formulated using model API, excipients (hypromellose, microcrystalline cellulose/MCC, pregelatinised starch/PGS, Sodium Stearyl Fumarate) whose batch numbers and vendor source were all kept constant. Moreover, identical formulation procedures were followed with one exception: pregelatinised starch (PGS) sourced from two vendors. The PGS from two different vendors were found to be similar concerning the certificate of analysis (CoA). However, the disintegration of tablets was found to be much dissimilar.

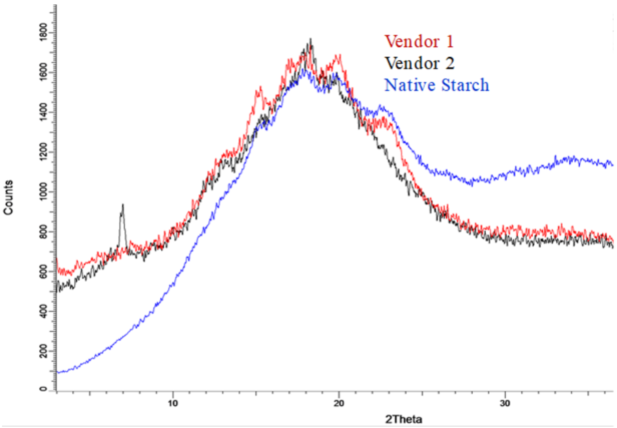


Figure 1: Comparison of the XRD patterns of native starch, PGS from Vendor 1 and PGS from Vendor 2

Figure 2: Properties of birefringence of Natural Starch (left), PGS from Vendor 1 (middle), PGS from Vendor 2 (right)

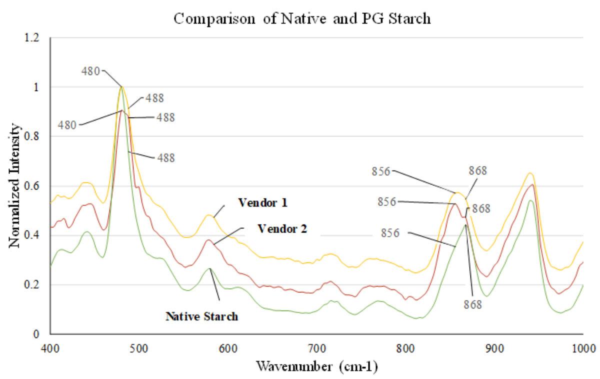


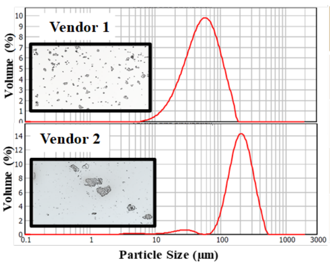
Figure 3: Comparison of the Raman Spectra of Native Starch, Pregelatinized Starch from Vendor 1 and Vendor 2

Powder X-Ray Diffraction (PXRD)

PXRD profiles of pregelatinised starch from two sources were collected on Bruker powder X-ray diffractometer (Model D8 ADVANCE); with theta-

Table 1: Viscosity Measurements of Pregelatinized Starch (n=3, in Centipoise, cP)

	Vendor 1	Vendor 2
Run 1	4.5	8.8
Run 2	4.7	9.5
Run 3	4.8	9.7
Average	4.7	9.3
SD	0.2	0.5
RSD	3%	5%



	Vendor 1	Vendor 2
D10	22.4	112.5
D50	53.5	202.0
D90	109.7	336.5
SSA	0.143 m²/g	0.0469 m²/g

Figure 4: Particle Size Distribution, Photomicrographs (as insert) and Three-tier distribution summary for PG Starch granules from two different vendors

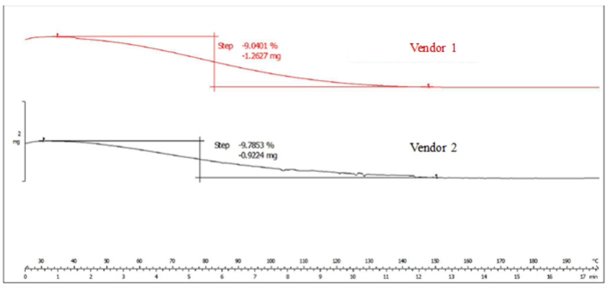


Figure 5: TG curves for pregelatinized starch from Vendor 1 and Vendor 2

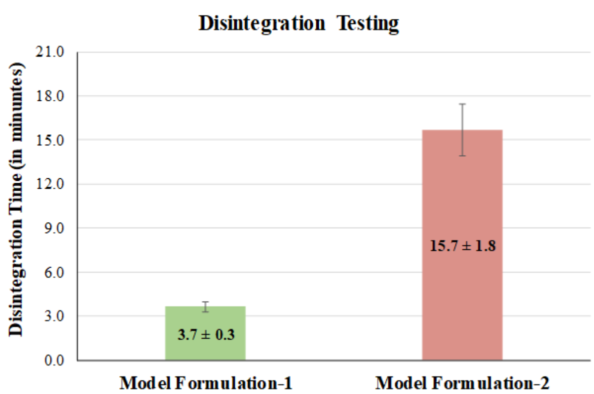


Figure 6: Comparison of disintegration time of formulations prepared using different pregelatinized starch from two vendors (n=6, Mean=±SD, error bars represent standard deviation)

theta geometry, a Copper anode (Kα1, λ = 1.5406 Å) and LynxEyeTM detector. The X-ray tube was operated at a voltage of 40 kV and a current of 40 mA. Each diffraction profile was collected in continuous mode and the scan range of 3° to 45° 2θ, with a step size of 0.01° 2θ and with a time per step of 0.1 sec. Top loading method was employed for sample preparation using PMMA (Poly-methyl methacrylate) sample holder (25 mm). The data processing was performed using Bruker Eva software. Parameters like sample grinding and rotation of holder were studied to understand the preferred orientation effects.

Particle Size Analyser (PSA)

Particle size distribution (volume-weighted) of PGS was measured by laser light diffraction (Malvern Mastersizer 2000, Malvern Instruments, Worcestershire, UK) using a dry sampling unit. Particle size calculation involved Mie theory approximation, and the following standard operating procedure (SOP) was used which were as follows, refractive index: 1.52, vibration feed rate: 25%, measurement time: 7 s, dispersive air pressure: 4 bar. Particle size distribution is characterised by the mass median diameter ($d_{0.5}$), i.e., the size in microns at which 50% of the sample is smaller, and 50% is larger, and the volume mean diameter (D4,3). Values presented are the average of at least three determinations.

Polarised Light Microscopy (PLM)

The polarised microscopic images of PGS were recorded with a CCD camera attached to the Nikon LV100 POL microscope and the data were analysed using the in-built NIS elements software. Few mg of samples were placed on a clean glass slide and were examined using PLM under various magnifications. To perform the birefringence experiments and to detect very weak birefringence, the λ/4 compensator is utilised. Optical, average particle size, particle shape and sample agglomerative properties of both the samples were compared.

Thermogravimetric Analyzer (TGA)

Thermal analysis was conducted using thermal gravimetric analysis (TGA) instrument (model: 851e/LF1100, Mettler-Toledo GmbH, Switzerland) equipped with a robotic arm (model: TSO801RO) for automated sampling. The TGA was conducted on pregelatinised starch from two vendors (sample weight was kept consistent ~10 mg). The total weight of each sample (accurately weighed into open 70 μL aluminium oxide crucibles using the TGA microbalance) was 10 ($\pm$ 1) mg. The powder sample was heated from 25 to 200 °C at a heating rate of 10 °C/min, under a nitrogen flush (10 mL per min). The instrument measured the change in mass and recorded the temperature profile and was calibrated using indium. Data were analysed using Stare Base Software for Windows 7(Mettler-Toledo GmbH, Switzerland)

Raman Spectroscopy

Raman spectra of PGS in triplicates for each sample were recorded with a Kaiser Raman RXN2 Analyzer (Kaiser Optical Systems, Inc., Ann Arbor, MI). The excitation wavelength of 785 nm actuated at 200 mW. Signals were detected by a Charge-Coupled Device (CCD) camera. Spectral acquisitions were made by using the iC Raman software (Kaiser Optical Systems). Spectral acquisitions were obtained in the range 100–1800 cm^{-1}, at a resolution of 1 cm^{-1} and each run contained an average of 75 scans as well as 10 s exposure time settings which resulted in a 15 min-long acquisition profile. The recorded Raman spectra were exported in .csv format and were processed in MS-Excel 2010.

Viscosity Measurements

The viscosity of the pregelatinised starches from two different vendors was studied over one hour. The samples (~5 grams) were taken in a 500 ml beaker to which 270 ml of distilled water was added, and the temperature was set at 25 °C whilst stirring. The samples were added slowly for one minute. Stirring was continued for two minutes, and the speed of the stirrer was adjusted to prevent the formation of vortex, agglomeration and incorporation of air. The viscosity of the prepared suspensions was determined by using a Brookfield viscometer (Model DV III, Brookfield Engineering Lab, Stoughton, MA, USA) at 100 rpm using a UL adapter and ULA spindle. Viscosity measurements were made in triplicate and were monitored regularly, and, the values were recorded after 60 mins.

Model Formulation

API, hypromellose, microcrystalline cellulose, pregelatinised starch were accurately weighed and passed through a 40-mesh screen to get uniform size particles and mixed in a blender for 15 min. The obtained blend was lubricated with sodium stearyl fumarate, and mixing was continued for a further 5 min. The only change being the pregelatinised starch procured from Vendor 1 (Model formulation-1) and Vendor 2 (labelled as model formulation-2). The resultant mixture was directly compressed into tablets by using 9 mm round flat-faced punch. Compression force was kept constant for both the formulations.

Disintegration Testing

As per the recommendations from the United States Pharmacopeia (USP) compendial test (701), product performance was assessed using disintegration testing which states the use of disks to understand the particle deaggregation as well as end-point detection in water as the immersion fluid (Davani, 2017). Average values obtained by testing six tablets disintegration were used to compare the performance of the model formulations.

RESULTS AND DISCUSSION

Indeed, it is well-recognised that the composition and structure of the starch control its properties (Whistler *et al.*, 2012). These influencing parameters can then be categorised into following: structural-level, molecular-level, particle-level, bulk-level as well as the impact of these parameters on the performance of the material.

Excipient Variability Analysis at Structural-Level using XRPD and PLM

The native starches possess two types of crystallinity (i) A-type, predominantly found in cereal starches (except high amylose variety), (ii) B-type, abundant in root and tuber starches (Whistler *et al.*, 2012). The X-ray diffractograms of native corn starch samples and pregelatinised samples from two vendors are shown in Figure 1. The native starch samples displayed high-intensity peaks at Bragg's angle (2θ) about 15°, 18°, and 23° that is representative of typical A-type crystalline pattern indicating its semi-crystalline nature. XRPD patterns for Vendor 1 displays peak at 2θ = 15°, 17°, 18° and 23° are visualised, suggesting that a certain degree of ordering of native corn starch is retained. On the other hand, XRPD patterns for Vendor 2 demonstrates halo and a peak at low 2θ = 5°. Based on these results, it can be interpreted that Vendor 1 provides a partially crystalline pregelatinised starch while Vendor 2 provides an amorphous grade of pregelatinised starch.

As illustrated in Figure 2. upon examination of native starch granules under a polarised light microscope (PLM), a typical birefringence with precise

Maltese crosses is visualised due to the hilum. Similarly, when pregelatinised starch from Vendor 1 is observed under PLM, majority of the granules were aggregated, slightly swollen but displayed a degree of birefringence, suggesting that the granules were not entirely disrupted, however, maintained a certain degree of crystallinity. Nevertheless, no birefringence can be observed in granules from Vendor 2, indicating that the ordered structure of starch was fully gelatinised and entirely disrupted (Liu *et al.*, 2017). The outcome of PLM measurements was consistent with the result of X-rays, showing the variability in the crystallinity between two materials.

Excipient Variability Analysis at Molecular-Level using Raman Spectroscopy

The gelatinisation process and its completion can be followed using vibrational spectroscopy (NIR, FTIR and Raman). The region between 450 to 500 cm^{-1} and 800 to 900 cm^{-1} is used to understand. Changes in peak width could be correlated to alteration in crystallinity/amorphicity (Mutungi *et al.*, 2012). Similarly, changes in peak intensity/position are indicative of different proportions of amylose or amylopectin, degree of crystallinity, starch retrogradation among the investigated samples. The peak at 480 cm^{-1} and 488 cm^{-1}, as well as 868 cm^{-1} and 856 cm^{-1}, are assigned to crystallinity changes as well as to starch retrogradation, respectively (Flores-Morales *et al.*, 2012). The high intensity of 488 cm^{-1} and 856 cm^{-1} indicates that Vendor 2 is fully pregelatinised while the opposing trend with Vendor 1 indicates the material has retrograded and recrystallised. Reiterating, starch is composed of alternating crystalline and amorphous regions which is evidenced with the illustrative maltese cross in PLM images. Peak width was found to be discriminative in the following chronological order native starch<Vendor 1<Vendor 2 indicating the Vendor 2 samples are amorphous, Figure 3. These results are in line with XRD and PLM. The correlation of starch granule size with the starch retrogradation indicates a completely varied gelatinisation pattern for Vendor 2.

Excipient Variability Analysis at Particle-Level using Laser Diffraction and Optical Microscopy

The pregelatinised starch particle sizes (in microns) obtained from both the vendors using laser diffraction are presented, see Figure 4. Particle size distributions were unimodal, narrow, however Vendor 1 showed d_{10} = 22 μm, d_{50} = 54 μm and d_{90} = 110 μm that were smaller in comparison with Vendor 2 which were d_{10} = 113 μm, d_{50} = 202 μm and d_{90} = 337 μm, respectively. There is a firm agreement between the particle sizes that are observed using microscopy and which is measured using laser diffraction.

Excipient Variability Analysis at Bulk-Level using TGA

Thermogravimetric analysis (TGA) offers detailed information on mass transition, temperature ranges typical of moisture/volatile release, and kinetic information. Representative thermograms are shown in Figure 5. The thermogram shows one weight-loss event/extensive endotherm observed between RT and ~140 °C, which is due to loss of water. This is consistent with both vendors demonstrating similarity in a loss on drying.

Excipient Variability and Correlation with Performance-Viscosity Measurements

The impact of both starch retrogradation, as well as particle size, was correlated to PGS performance by measuring the viscosity build-up as a deterministic parameter for 60 mins. That is, it is vital to understand the rheological or flow behaviour to compare the binder/disintegrant efficiency of pregelatinised starch. Since PGS demonstrates shear thinning behaviour where the viscosity decreases along when the shear rate increases. Hence the shear rate was kept constant for PGS obtained from both the vendors.

The efficiency of PGS from both the vendors has been evaluated as a function of the viscosity. The 2% w/v aqueous dispersion maintained at a temperature of 25 °C for Vendor 1 and Vendor 2 had a viscosity of 4.7 $\pm$ 0.2 cP and 9.3 $\pm$ 0.5 cP, respectively, for details refer Table 1. That is, Vendor 2 demonstrated higher viscosity than Vendor 1, which could retard dissolution due to viscosity build-up (Zámostný *et al.*, 2012). Additionally, formulations were prepared and subjected to disintegration testing.

Excipient Variability and Correlation with Performance-Disintegration Testing

Two formulations were prepared to test the hypothesis mentioned above using the same components (API and excipients), with one exception, which is PG starch from two different vendors. In one formulation, PG starch from Vendor 1 (termed model Formulation-1) was used while in the other formulation Vendor 2 (termed model Formulation-2) was employed. Since the API in the formulations is categorised as BCS class 3, the use of disintegration as a surrogate test to dissolution is employed (Nickerson *et al.*, 2018). Disintegration results for both the model formulation tablets manufactured using different PG starch values are provided in Figure 6. Model formulation-1 was found to comply with the in-house specification, which is, disintegration time of <4.5 mins while the model formulation-2 show a DT of ~15 mins. Results indicate that the mean time taken for the disintegration of model formulation-2 is higher in comparison with the other formulation.

Correlation of Physical Characterisation with Performance Tests

Pregelatinized starch is a modified corn starch which exhibits high water holding capacity. Furthermore, it is a versatile excipient that is pH-independent disintegrant, diluent, stabiliser for water-sensitive drugs etc. Physico-chemical characterisation indicated that Vendor 1 contained smaller particles, birefringence, crystallinity similar to native starch. In contrast, Vendor 2 was found to be more like an amorphous fully pregelatinised starch. In order to correlate the material property, two performance measurement tools like viscosity test for PG starch and disintegration test for model formulations were carried out. The viscosity of Vendor 2 resembled the specification of pregelatinised material in line with the specifications, while for Vendor 1, it was close to native starch.

Similarly, the disintegration test was evaluated for a potential correlation. The disintegration time for the model formulation-1 was within the specification of <4.5 mins while for model formulation-2 it was ~15 mins. The trends observed with disintegration, mirror those observed with the physicochemical measurements and viscosity measurements. It has been well-recognised that full pregelatinization makes the excipient soluble in cold water and the PGS would behave as a binder while it would be ineffective as a disintegrant. Zamostny and Majerova explained that the soluble portion of pregelatinised starch would swell (related to the cold water solubility of PG starch) and it could inhibit the caffeine diffusivity (Zámostný *et al.*, 2012). That is, the PG starch from Vendor 2 displayed binding property, whereas PG starch from Vendor 1 displayed disintegrant property.

CONCLUSIONS

A primary objective of this study was to evaluate the functionality of pregelatinised starch from two different vendors as part of 'Alternate Sourcing Strategy". Albeit the monograph provides tests of purity and identification, it does not provide any assurance on the materials' functional characteristics or their influence on the performance of the product. Results of this study confirmed that the differences in crystallinity, starch retrogradation, particle size within the pharmacopoeial specifications could display significant differences in their influence on PG starch viscosity and tablet disintegration. Moreover, it is demonstrated that minimal variations in the properties of the pregelatinised starch, which is used as a disintegrant or binder in drug product formulations, can significantly alter the release profile of active pharmaceutical ingredient. That is, a thorough characterisation of the material properties and its correlation to product performance is considered imperative. In conclusion, functional similarity rather than pharmacopoeial similarity is considered as the key criterion for the projects involving "Alternate Sourcing Strategy".

Acknowledgement

We are so much obliged to Sanofi and Dr Praveen Khullar, Head of Global Development Centre, Sanofi Synthelabo (India) Private Limited, Goa and Dy. Head, Scientific and Technical Services, for providing the facilities to carry out the work mentioned above. We also thank Dr A. Rose Venis, Associate Professor, Department of Chemistry, St. Joseph's College (affiliated to Bharathidasan University), Trichirappalli for providing guidance and the needful support.

Conflicts Of Interest

All authors declare that there are no conflicts of interest for this study.

Funding Support

All authors declare that there is no funding support for this study.

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Original Paper | Published: 10 October 2020

Structural Elucidation and Position Identification of Cu(II) ion in Hexaaquazinc(diaquabismalonto)zincate: Single Crystal EPR and Optical Studies

A. Parthiban, K. Arun Prasath Lingam, S. Mangalaraj, P. Muthukrishnan & M. Surendiran

Journal of Cluster Science **32**, 1401–1409 (2021) | Cite this article

Abstract

Single crystal electron paramagnetic resonance (EPR), powder X-ray diffraction (XRD),

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M. S.

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Memorandum of Understanding

This Memorandum of Understanding (herein after referred to as the "MOU") is made at Tiruchirappalli on 18th day of January 2016 between Educational Institution located at College Road, near Chatram Bus Stand, Tiruchirappalli represented herein by Fr. F. Andrew, Principal of St. Joseph's College (herein after called "Educational Institution") and the Tamil Nadu Pollution Control Board(TNPCB) located at 25,SIDCO Industrial Estate, Thuvakudi, Tiruchirappalli -15 represented herein by its District Environmental Engineer, Tmt. R. Lakshmi (herein after called "TNPCB").

TNPCB is interested in establishing Continuous Ambient Air Quality Monitoring Stations (CAAQMS) in Tamilnadu with the participation of the Educational Institutions at 25 locations identified jointly by the TNPCB and Educational Institutions.

18/1/16
Rev. Dr. F. ANDREW, S.J.
PRINCIPAL
ST. JOSEPH'S COLLEGE (Autonomous)
TIRUCHIRAPPALLI - 620 002

18/01/2016
DISTRICT ENVIRONMENTAL ENGINEER
TAMIL NADU POLLUTION CONTROL BOARD
TIRUCHIRAPPALLI

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1.0 Role of TNPCB

i. All the Instruments/Equipments will be purchased by calling Open Tenders by TNPCB.
ii. They will be established in the identified places / stations by the TNPCB.
iii. The rent if any fixed for the building identified for CAAQM Station will be paid by TNPCB after making agreement with the owner of the building for a minimum period of 5 years, which will be renewed by another 5 years or more as per the PWD rates at the option of TNPCB.
iv. The initial three phase power supply provisions and telephone connection with internet facility for CAAQM Stations will be arranged by TNPCB. Internal wiring for the stations will also be arranged by TNPCB as per the requirement given by the O&M contractor.
v. After complete establishment of the stations by the supplier, the stations will be handed over to the O & M contractor by getting indemnity bond prescribed for this purpose in the tender documents.
vi. TNPCB will issue Operation and Maintenance Contract (O & M) to the supplier to maintain the CAAQM Stations initially for 5 years which may be renewed to another 10 years (in 5 years package) or up to the life period of the analyser, whichever is later at the option of TNPCB.
vii. The O&M payment will be made by TNPCB after getting necessary certificates from the respective Educational Institutions as prescribed in the tender documents and in the O&M conditions along with invoice of the contractor.

2.0 Role of Educational Institutions

i. Educational Institutions will identify suitable buildings for the proposed CAAQM Stations in coordination with the concerned District Environmental Engineers of TNPCB and fix rent if necessary as per the PWD norms.

18/1/16

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DISTRICT ENVIRONMENTAL ENGINEER
TAMIL NADU POLLUTION CONTROL BOA
TIRUCHIRAPPALLI

ii. Educational Institutions will identify a qualified staff for daily maintenance of the CAAQM Stations to the satisfaction of O & M contractor. The identified staff will be responsible to the O & M contractor for the purpose of O&M and the O&M contractor will pay to the Educational Institution the salary and other benefits for the employee. The Identified employee shall be entirely deputed to the O&M contractor The O&M contractor will provide enough training to the staff for daily maintenance of the stations. After getting trained, the staff should maintain the station in co-ordination with the O&M contractor. They shall be deemed to be the employees of the Educational institution and there shall not exist any employer employee relationship between TNPCB and the workers employed by the Educational Institution

iii. Each station will be connected to the respective Educational Institutions by Internet by providing necessary softwares, computers and printers by the O & M contractor so as to monitor the stations whether they are working or not and also get the reports as per the formats prescribed for this purpose. The internet services only will be provided by the Educational Institutions. All 25 Stations will also be connected to the CAC, TNPCB Chennai, and IIT, Chennai for data analysis.

iv. The validated data generated by the O & M contractor from the stations will be utilized by Educational Institutions for source apportionment studies and suggest necessary action plans for abatement of pollution in the particular station by using air monitoring models. The TNPCB and other experts will be associated in the project for this purpose as may be required for the studies.

v. Educational Institutions must carry out periodic inspections of the CAAQM Stations and report whether O & M contractor maintaining the instruments as per the O & M conditions and validated data are obtained by scheduled/periodic calibration of the analysers. For this purpose, the inspection format prescribed should be maintained and furnished by the educational institutions along with the certified invoice of the O & M contractor for making payment by TNPCB.

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TAMIL NADU POLLUTION CONTROL BOARD
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vi. The suggested action plans if any from the Educational Institution shall be sent periodically to TNPCB for the real time data obtained.

3.0 Role of Operation and Maintenance (O&M) Contractor

i. The O & M contractor will maintain the stations under the direct supervision of respective Educational Institutions and TNPCB nominees identified for the stations in all aspects like periodic calibration of analyzers, security, housekeeping and insurance.
ii. All the recurring expenditure like AMC for instruments / analysers / A.C / UPS, electricity charges, insurance, security expenditure, manpower; housekeeping, monthly payment of internet and telephone charges etc. will be incurred by the O&M contractor during the O & M period.

4.0. The Educational Institution and O&M contractor shall mutually agree to carry out the above activities of air quality monitoring station identified for at least for 10 years.

5.0 The data generated is the property of the TNPCB and it could be used by the Educational Institution for research purposes only with the consent of TNPCB in writing.

6.0 Publication

6.1 TNPCB recognizes that under its policy, the Educational Institution shall have the right, at its discretion to release information or to publish any material resulting from the research provided that such material does not include any TNPCB's Confidential Information and that such publication would not disclose or otherwise jeopardize any potential patent rights, in technology developed hereunder. The data before publication shall be sent to IIT for validation.

6.2 The Educational Institution should ensure the safety of the data.

18/1/16
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18/01/2016
DISTRICT ENVIRONMENTAL ENGINE
TAMIL NADU POLLUTION CONTROL B
TIRUCHIRAPPALLI

6.3 The TNPCB will be given full credit and acknowledgment for any support provided to the Educational Institution in any publication resulting from this Agreement, unless requested otherwise by TNPCB.

7.0 Intellectual Property Rights

7.1 TNPCB shall retain ownership of all Intellectual property developed and / or purchased by TNPCB or on TNPCB'S behalf prior to this Agreement and to all Intellectual property developed and / or purchased by or on TNPCB's behalf without the active participation of the Educational Institution.

7.2 Except for the rights expressly granted to the Educational Institution under this Agreement, the TNPCB will retain all right, title and interest in and to the Intellectual property and proprietary rights.

8.0 Duration of Agreement

This Agreement is effective for 10 years and this can be renewed for a further period of 5 years and so on at the option of TNPCB.

9.0 Surrender of Materials on Cessation

Educational Institution agrees to deliver to TNPCB, and not to keep or deliver to any other person or entity, on the date of the cessation of this agreement, all documents and things in cessation of this agreement, including but not limited to, Confidential Information. If documents and things pertaining to the business of TNPCB or originating with TNPCB come into Educational Institution's possession after his/her cessation from the PROJECT, he/she will promptly deliver them to TNPCB.

18/01/2016

DISTRICT ENVIRONMENTAL ENGINEER
TAMIL NADU POLLUTION CONTROL BOARD
TIRUCHIRAPPALLI

18/1/16

Rev. Dr. F. ANDREW, S.J.
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ST. JOSEPH'S COLLEGE (Autonomous)
TIRUCHIRAPPALLI - 620 002

10.0 Indemnity

10.1 The O & M contractor is responsible for quality of data generated from CAAQM Station.

10.2 To the maximum extent permitted by law, TNPCB hereby agrees to indemnify, defend, and hold harmless the Educational Institution and its present and former officers, directors, governing board members, employees, agents, and students from any claim, loss, cost, expense, damage or liability of any kind, including reasonable attorney's fees and expenses, arising out of or connected with its use of the Background Intellectual Property or Jointly-Owned Intellectual Property.

Tamil Nadu Pollution Control Board

18/01/2016

Tmt.R.Lakshmi
District Environmental Engineer
Tamilnadu Pollution Control Board
25,SIDCO Industrial Estate
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DISTRICT ENVIRONMENTAL ENGINEER
TAMIL NADU POLLUTION CONTROL BOARD
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Educational Institution

18/1/16

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Memorandum of Understanding

Between

St. Joseph's College, Tiruchirappalli

And

International Skill Development Corporation (ISDC)

MEMORANDUM OF UNDERSTANDING

This Memorandum of Understanding is entered into at Madurai on this 18 day of December in the Year 2019.

BETWEEN:

ISDC Projects India Pvt. Ltd, trading as ISDC - International Skill Development Corporation, and having its Regional Office at Lakhsmi Narayan Complex, Palace Road, Vasanth Nagar, Bengaluru, Karnataka - 560052 represented by its Head- Institutional Partnerships – Mr. Daya Murthy, hereinafter referred to as "ISDC" or the FIRST PARTY

AND

St. Joseph's College is a Bharathidasan University-affiliated college located in College Main Road, Annamalai Nagar, Teppakulam, Tiruchirappalli, Tamil Nadu 620002. The College is autonomous and has an independent examination and recognition system; represented by its Principal Rev. Dr. M. Arockiasamy Xavier, S.J, hereinafter referred to as "St. Joseph College" or the SECOND PARTY

Both parties as above have expressed a desire of entering into a Memorandum of Understanding to meet their respective objectives, which are set out herein below.

WHEREAS

a. The First Party- ISDC, a Skill Development& Education Company having expertise in Professional and Vocational Education and is interested in associating with the Second Party to develop, promote and deliver B.Com (Honours) Integrated with ACCA, which is accredited by the ACCA (Association of Chartered Certified Accountants), UK Professional Body in Chartered Accountants.

b. The Second Party on its part is interested in associating with First Party for using their expertise to develop, promote and deliver the course mentioned above as the Undergraduate program of St. Joseph College with their Academic Autonomy and which leads to Advanced Diploma from ACCA to the students enrolled with the Second Party along with their Undergraduate degree from St. Joseph College.

The purpose of this Memorandum of Understanding is to set forth the terms and conditions under which the Parties to this understanding shall conduct themselves during the subsistence of the Memorandum.

This Memorandum of Understanding (**"MOU"**) is not intended to be, legally binding except as specifically set out below.

1. The First Party shall support the Second Party to develop the undergraduate course mentioned in the First Part (Part A) of this MoU. The Students enrolled for this course/program leads to the additional qualification / Membership from the respective professional organization as mentioned above (Part B).

2. It is the responsibility of the Second Party to get the necessary approvals for running the program at St. Joseph College. The final approved syllabus by the Board of Studies and Academic Council of the institution for the Course/ Program with the exemptions availed from professional body through program accreditation process will be given as Annexure 1.

3. The First party can use the name of the St. Joseph College for promoting the above program in advertisements and other modes of communications. The Promotion of the said program has to be taken-care jointly by the parties.

4. The admission criteria and the number of seats for the above said programs are fixed by the Second Party in consultation with First Party.

5. The First Party facilitates train the trainer program for 8 hours per paper of ACCA to the Faculties of the Second Party, as per mutually agreed time schedule.

6. The First Party provides Electronic Copy as well as limited printed copy for reference of relevant learning materials (For the subjects of integrated curriculum of professional body only) to the students enrolled for the above said Course/Program

7. The First Party provides training support to the students enrolled for the above program and the number of hours per subject / paper/module is given as Annexure 2.

8. All responsibilities regarding registration of the students with professional body should be dealt with, by the First Party. The students have to follow the Rules and Regulations of Respective Professional body to appear the examination and pursue the qualification/ membership and designation.

9. The relevant fee to professional bodies has to be paid by the students directly as per the rules and regulations set by the professional bodies time to time as per Annexure 3.

10. Out of the Tuition Fees Collected, the Second Party has to make the necessary payment to the First party as per the table given below:

Course/Program	Amount
B.Com (Honours) Integrated with ACCA	INR 36,000 Per Student for entire three years
First Installment - After admission confirmation and course commencement in 2020	INR 18,000
Second Installment – Before February 28, 2021	INR 18,000

This is for the Batch of 2020-2021 while the same will continue for subsequent fresh batches. The above fee will be a part of the college fee.

11. The complete list of Students of the course shall be provided to the First Party by the Second Party. Changes in Students, if any, shall be communicated by the representative of the Second Party to the First Party immediately.

12. It is intended that the terms of this MoU will remain in force for an initial period of three years set out above i.e., the completion of First Batch or is otherwise terminated in accordance with the provisions of Clause 13. The MoU can be extended for further periods after the expiry of three years upon the parties mutually agreeing such extension in writing. The terms of this Memorandum may be modified at any time by both parties on mutual consent.

13. Either party shall be entitled to terminate the MOU on 60 days' notice. The MoU will automatically terminate:

- Any potential Binding Agreement would be unenforceable, void or illegal due to any statutory or regulatory requirements; or
- Terminates any Binding Agreement for cause.
- In the event of the termination of the agreement, the First Party agrees to complete the existing batches on agreed terms.

14. All intellectual property created by a party in connection with the collaboration shall remain the property of that party. The parties agree that any material jointly created by the parties for the collaboration shall be jointly owned (in equal proportions) by the parties, unless otherwise agreed in writing.

15. Where the collaboration reasonably requires the use by one party of intellectual property that is owned by the other party (the **"IPR Owner"**), the IPR Owner will license such rights to the other party on a non-exclusive basis, without the right to sub-license, solely for the purpose and to the extent necessary in connection with the collaboration. Any such license will automatically terminate when the MoU is terminated.

16. The college will comply by providing the required documents for ACCA Accreditation, academic program guide with details of the integrated syllabi and sample question papers.

17. For the purposes of this MoU, **"Confidential Information"** shall be all information of a confidential nature (whether written or oral) concerning the business and affairs of either party which is obtained or received as a result of the discussions leading up to, the entering into or the performance of this MoU, including financial information, training & learning material, trade secrets, college lists, trade and commercial details and computer software and databases, the contents of all reports and documentation prepared by either party or on its behalf and any other information of a confidential nature designated by a party as confidential; Each of the parties shall at all times while this MoU remains in force and after it has terminated, keep confidential the Confidential Information except where:

- The Confidential Information was already lawfully known, or became lawfully known to either of the parties independently;
- Disclosure or use is necessary by either of the parties (including their employees, agents and sub-contractors) for the proper and effective performance of this MoU;
- Disclosure is required by law to any government, governmental department, agency, regulatory or fiscal body or authority (whether national or foreign) and their authorized agents (including professional advisers);
- The Confidential Information is disclosed, in the case of ISDC, to another member of the ISDC group of companies;
- Each party undertakes to the other that it will not disclose or make use of, for its own benefit, any of the Confidential Information of that other party.

18. All disputes and differences of any kind whatever arising out of or in connection with this MoU shall be referred to the arbitration, and the final decision of an arbitrator Jointly appointed by both the parties to be agreed upon and appointed by both the parties, or in case of disagreement as to the appointment of a single arbitrator, two arbitrators, one to be appointed by each party and if there are two arbitrators, they shall before taking upon themselves the burden of reference appoint a third arbitrator

who shall act as Presiding Arbitrator. This submission to the arbitrators shall be deemed to be a submission to arbitration within the meaning of the **Arbitration and Conciliation Act, 1996**, or any statutory modification thereof. The award of the arbitrator or arbitrators as the case may be, shall be final and binding on the parties.

19. This MoU does not bear any legal action status. However, In case of any disputes not settled due to arbitration it will be subject to the courts of Bangalore Jurisdiction.

Each party hereby confirms its agreement to the terms contained in this MOU on this 18 day of December, 2019.

On behalf of

St. Joseph College
Tiruchirappalli

18\12\19

Rev. Dr. M. Arockiasamy Xavier, S.J

Principal

ISDC

18|12|2019

Daya Murthy

Head – Institutional Partnership

IN WITNESS WHEREOF, the parties hereto have caused this Memorandum of Understanding to be executed as of the first date set forth above.

On this 18 day of December 2019.

Witness:

Annexure 1

"The Approved SYLLABUS from BOS constituted by St. Joseph College, Tiruchirappalli comes here"

Annexure 2

B.Com (Honours) ACCA Master Classes by ISDC Trainers

Module	Sessions
F7 Financial Reporting	40 Hrs
F8 Audit and Assurance	40 Hrs
F9 Financial Management	40 Hrs

Annexure 3

ACCA Exam fees		
Exams	ACCA Papers	GBP
Exempted	F1 (Accountant in Business)	0
Exempted	F2 (Management Accounting)	0
Exempted	F3 (Financial Accounting)	0
Exempted	F4 (Corporate and Business Law)	0
Exempted	F5 (Performance Management)	0
Exempted	F6 (Taxation)	0
Exam	F7 (Financial Reporting)	106
Exam	F8 (Audit and Assurance)	106
Exam	F9 (Financial Management)	106
	Total	318

*The mentioned exam fees are as per standard entry

** Annual Subscription payable separately 112 GBP

Please Note:

The ACCA Fess can be found on the URL given below and it is subject to the discretion of ACCA.

http://www.accaglobal.com/in/en/qualifications/accountancy-career/fees/fees-charges.html?countrycode=India

In addition to the above Fees, the students will have to pay 20 GBP towards Initial Registration
The Exemption Fees for the exempted papers from ACCA is waived off. [Benefit for the student is approx. 600 GBP]
There is no wavier for Annual Subscription and Examinations Fees.

Think Ahead ACCA

Memorandum of Understanding

Commencement Date: the last date of signature **by both parties of this document being 27th – February 2020**

Between **St. Joseph's College (Autonomous), Tiruchirappalli-620 002, Tamilnadu, India.**

and

Association of Chartered Certified Accountants, incorporated by Royal Charter (number RC000732) of The Adelphi, 1–11 John Adam Street, London WC2N 6AU ("ACCA")

1. Background

1.1. This Memorandum of Understanding (MOU) made between the St. Joseph's College, Tiruchirappalli and ACCA provides for the establishment of a mutual co-operation between these two institutions.

1.2. Recognising the value of promoting mutual co-operation for the advancement of their respective members and the accountancy and tax professions, St. Joseph's College, Tiruchirappalli and ACCA agree to the following terms as set out below.

2. Purpose

2.1. The purpose of this MoU is to set out the understanding between the Parties without any intention to create legal relations, rather in the spirit of mutual cooperation. Any collaborative ventures that may bind the Parties are subject to separate contractual arrangements. The purpose of this MoU is to explore opportunities for collaboration to their mutual benefit, to set out the respective roles and responsibilities of the Parties in working cooperatively to further each other's and their mutual interests.

2.2. No funding will be required from either Party except as mutually agreed from time to time. The Parties agree that all financial arrangements will be negotiated for each specific case prior to commencement of the activity and will depend on the availability of funds.

3. Duration and Termination

3.1. This MOU will be deemed to have come into force and effect on the Commencement Date and will continue for a period of three (3) years from that date. The parties agree to review this MOU annually. This MOU may be terminated at any time during its term by either ACCA or St. Joseph's College, Tiruchirappalli upon three (3) months' prior written notice to the other.

3.2. Either party may terminate this MOU by immediate written notice to the other party in the event of an occurrence of any act, omission or conduct which is deemed by the Terminating Party, at all times acting reasonably, to undermine, jeopardise or damage the professional reputation of the Terminating Party.

3.3. This MOU may be amended upon the mutual written agreement of both parties.

4. Specific areas of co-operation

ACCA undertakes to work together with St. Joseph's College, Tiruchirappalli in the following areas:

4.1. Provide access to ACCA professional development and learning resources

4.2. Learning and development initiatives - e.g. competencies, skills and training mapping, etc.

4.3. Jointly develop strategies and marketing campaigns to promote the global collaboration between St. Joseph's College, Tiruchirappalli and ACCA

4.4. Work collaboratively and carry out joint engagement and outreach to promote and create awareness about ACCA trainings, Qualification, ACCA-X, master's programme and other relevant trainings

4.5. Joint partnership programmes supporting the training and development of students to prepare them for successful careers in accountancy and finance;

4.6. Explore and create new pathways for students to further their education and enhance their careers in accountancy and finance;

4.7. Enhance the employability of students through capacity building initiatives;

4.8. Tap into one another's networks of professional and industry contacts to support the above objectives

5. Other areas of co-operation

St. Joseph's College, Tiruchirappalli and ACCA will also explore other potential areas where co-operation would be of mutual benefit.

6. Confidentiality

The parties agree to keep confidential any information which is disclosed or obtained and which is not publicly available or already known and not to disclose such information to third parties, otherwise than in accordance with the consent of the other party or as required by law or any relevant regulatory authority.

7. Anti-bribery and Anti-Slavery Measures

7.1. Each Party undertakes to the other to comply with applicable laws, regulations, codes and sanctions relating to anti-bribery and anti-corruption and each other's respective anti-bribery and gift and hospitality policies (where applicable) as may be amended from time to time, copies of which will be provided on written request.

7.2. Each party shall ensure that it and any persons or subcontractors involved in the delivery of the objectives of this MOU shall comply with all applicable anti-slavery and human trafficking laws, statutes, regulations and codes from time to time in force.

7.3. Breach of this part 7 shall be deemed to be a material breach and in case either party breaches this part 7, the non-breaching party may terminate the MOU immediately by written notice.

8. Intellectual Property Rights

8.1. All intellectual property content that is developed jointly by the Parties after the commencement of this MOU will be jointly owned by the Parties and subject to any other terms and conditions that will be worked out on a case-to-case basis, as may be agreed upon in writing.

8.2. Any intellectual property that is owned by either of the Parties, pre dating this agreement, and is exploited or modified under this MOU, will remain the sole property of that Party. It will therefore not be reproduced or transmitted in any other form or by any other means, electronic or mechanical, including photocopying, recording on any storage or retrieval system, without the prior permission and written consent of the intellectual property owner.

8.3. Use of logos, trademarks, intellectual property, copyright materials, etc. will be in accordance with each organisation's guidelines. Neither Party shall use, nor permit any person or entity to use the name, logo (or any variation thereof), intellectual property, copyright materials, etc. of the other party without first obtaining the other Party's written consent.

9. Assignment

Unless this MOU expressly states otherwise, no right or obligation arising under this MOU may be assigned, transferred or otherwise disposed of, in whole or in part, without the prior written agreement of the parties.

10. Form of Understanding

10.1. This MOU outlines the areas of co-operation that have been agreed between St. Joseph's College, Tiruchirappalli and ACCA, however nothing in this MOU should be construed as creating legal obligations between the two parties, except for clauses 6 (Confidentiality), 7 (Anti-bribery Measures) and 8 (Intellectual Property). This MOU supersedes any previous agreement between the parties relating to its subject matter.

11. Good Faith

11.1. In entering into this MOU, the parties recognise that it is impractical to make provisions for every contingency that may arise during the course of the MOU.

11.2. Accordingly, the parties declare it to be their intention that this MOU shall operate between them in accordance with the principles of good faith, with fairness and without detriment to the interests of anyone and if any dispute arises, the parties shall use commercially reasonable endeavours to agree upon such action as may be necessary and equitable to remove or resolve the cause or causes of the same.

St. Joseph's College, Tiruchirappalli

Name: Rev.Dr.M. Arockiasamy Xavier, S.J.,

Position: Principal

Date: 27th February 2020

ST. JOSEPH'S COLLEGE (Autonomous) ★ Tiruchirappalli-2 ★

Signed for and on behalf of Association of Chartered Certified Accountants

Name: Mohammed Sajid Khan

Position: Head of International Development

Date: 27th February 2020

In the presence of

One representative
From St. Joseph's College
Tiruchirappalli

Dr. F.R.ALEXANDER PRAVIN DURAI,
M.Com.,MBA.,M.Phil.,Ph.D.,
Head & Associate Professor
Department of Commerce
St. Joseph's College (Autonomous)
Tiruchirappalli-620 002.

B Saravanakumar

Regional Manager – South, ACCA

தமிழாய்வுத்துறை, வரலாற்றுத்துறை

தூய வளனார் கல்லூரி (தன்னாட்சி)

திருச்சிராப்பள்ளி – 620 002

ஆசா அறக்கட்டளைச் சொற்பொழிவு & பரிசளிப்பு விழா

18.03.2021, வியாழக்கிழமை, முற்பகல் 11.00 மணி

இடம் : SAIL அரங்கம்

வரவேற்புரை	: முனைவர் **கி.லிங்கம்மாள்,** துறைத்தலைவர்(பொ), வரலாற்றுத்துறை
முன்னிலை	: திருமதி **கேத்தரின் ஆரோக்கியசாமி**
தலைமை	: அருள்முனைவர் **ம. ஆரோக்கியசாமி சேவியர்,** சே.ச. முதல்வர், தூய வளனார் கல்லூரி
வாழ்த்துரை	: **சூ. மார்ட்டின்,** B.Com., M.L.
அறக்கட்டளை அறிமுக உரை	: முனைவர் **ஆ. இராஜாத்தி**
சிறப்புரை	: முனைவர் **ஆ. குழந்தைசாமி** தமிழ் மொழியியல் துறை, சென்னைப் பல்கலைக்கழகம், சென்னை
பொருண்மை	: **'வீரமாமுனிவரின் ஆளுமைகள்'**

பரிசு வழங்குதல்

நன்றியுரை	: முனைவர் **போ. ஜான்சன்**
நிகழ்ச்சி நெறியாள்கை	: திருமிகு **பா. எழில் செல்வன்** முனைவர் பட்ட ஆய்வாளர்

இனிதே அழைக்கும்

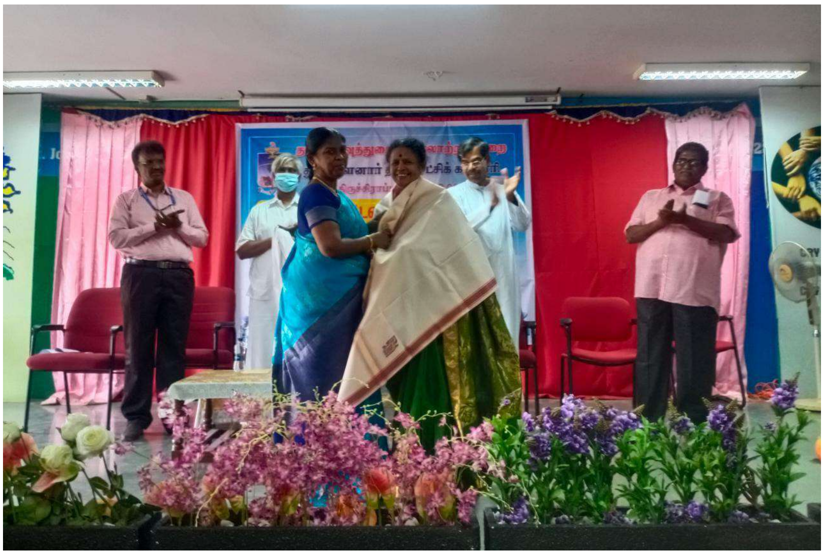
கல்லூரி முதல்வர் & துறைப் பேராசிரியர்கள்

Dr. G. BESCHI
Associate Professor & Head
Department of Tamil
St. Joseph's College (Autonomous)
Tiruchirappalli-620 002.

ஆசா அறக்கட்டளைச் சொற்பொழிவு மற்றும் பரிசளிப்பு - 18.03.2021

தமிழாய்வுத் துறை மற்றும் வரலாற்றுத் துறை இணைந்து ஆசா அறக்கட்டளைச் சொற்பொழிவு மற்றும் பரிசளிப்பு விழா நடைபெற்றது. இறை வணக்கம் மற்றும் தமிழ்த்தாய் வாழ்த்துடன் தொடங்கிய இவ்விழாவில் வரலாற்றுத்துறை பொறுப்புத் துறைத்தலைவர் முனைவர் லிங்கம்மாள் வரவேற்புரையாற்றினார். இக்கல்வியாண்டிற்கான அறக்கட்டளை பொறுப்பாளர் முனைவர் இராசாத்தி அறக்கட்டளை அறிமுக உரையாற்றினார். கல்லூரி முதல்வர் **அருள்முனைவர் ம. சேவியர் ஆரோக்கியசாமி, சே.ச.** அவர்கள் தலைமையில் ஏற்பாடு செய்யப்பட்டுள்ள இவ்விழாவில் தமிழாய்வுத்துறைத் தலைவர் **முனைவர் ஞா.பெஸ்கி,** வழக்கறிஞர் **சூ.மார்டின்** ஆகியோர் முன்னிலை வகித்தனர். அனைத்துக் கல்லூரி மாணவர்களுக்காக 16.03.2021 அன்று நடத்தப்பட்ட போட்டிகளில் பரிசு பெற்ற திருச்சி தேசியக் கல்லூரி, சென்னை லயோலா கல்லூரி, வேட்டவலம் லயோலா கல்லூரி, புதுக்கோட்டை மாட்சிமை தங்கிய மன்னர் கல்லூரி மாணவர்களுக்குப் பரிசுகளும், சான்றிதழ்களும் வழங்கப்பட்டன. சென்னைப் பல்கலைக்கழகத் தமிழ் இலக்கியத்துறைப் பேராசிரியர் **முனைவர் ஆ.குழந்தை** வீரமாமுனிவரின் ஆளுமைகள் என்கிற மையப்பொருளில் சிறப்புரை ஆற்றினார். முனைவர் போ.ஜான்சன் நன்றியுரையாற்றினார். ஆய்வு மாணவர் பா.எழில் செல்வன் நிகழ்வுகளைத் தொகுத்து வழங்கினார். இவ்விழாவில் வேட்டவலம் லயோலா கல்லூரி, திருச்சி தேசிய கல்லூரி மாணவர்கள் உட்பட, தூய வளனார் கல்லூரி தமிழ் மற்றும் வரலாற்றுத்துறை இளங்கலை, முதுகலை மாணவர்கள், பேராசிரியர்கள், தமிழ் ஆர்வலர்கள், வழக்கறிஞர்கள் பங்கேற்றனர்.

ஒளிப்படம்



Dr. G. BESCHI
Associate Professor & Head
Department of Tamil
St. Joseph's College (Autonomous)
Tiruchirappalli-620 002.

செய்தித்தாள் பதிப்பு

போர் புதியமுரசு 4

திருச்சி தூய வளனார் கல்லூரியில்

ஆசா அறக்கட்டளை சொற்பொழிவு மற்றும் பரிசளிப்பு விழா

திருச்சி,மார்ச்.22: திருச்சி தூய வளனார் கல்லூரித் தமிழாய்வுத் துறை மற்றும் வரலாற்றுத் துறை இணைந்து மேனாள் சட்டமன்ற உறுப்பினர் ஆரோக்கியசாமி அவர்களின் நினைவாக அவரது துணைவியார் கேத்தரின் ஆரோக்கியசாமி அவர்கள் நிறுவியிருக்கிற ஆசா அறக்கட்டளைச் சொற்பொழிவு மற்றும் பரிசளிப்பு விழா நடைபெற்றது. இறைவணக்கம் மற்றும் தமிழ்த்தாய் வாழ்த்துடன் நிகழ்ச்சி தொடங்கியது. இவ்விழாவிற்கு கல்லூரி முதல்வர் அருள்தந்தை சேவியர் ஆரோக்கியசாமி அவர்கள் தலைமையில் ஏற்பாடு செய்யப்பட்டுள்ள இவ்விழாவில் தமிழாய்வுத்துறைத் தலைவர் முனைவர். ஞா.பெஸ்கி, வழக்கறிஞர் சூ.மார்டின் ஆகியோர் முன்னிலை வகித்தனர். இவ்விழாவில் வரலாற்றுத்துறை பொறுப்புத்துறைத் தலைவர் முனைவர். லிங்கம்மாள் வரவேற்புரையாற்றினார். இக்கல்வியாண்டிற்கான அறக்கட்டளை பொறுப்பாளர் முனைவர்.இராசாத்தி அறக்கட்டளை அறிமுக உரையாற்றினார். அனைத்துக் கல்லூரி மாணவர்களுக்காக நடத்தப்பட்ட போட்டிகளில் பரிசு பெற்ற திருச்சி தேசியக் கல்லூரி, சென்னை லயோலா கல்லூரி, வேட்டவலம் லயோலா கல்லூரி, புதுக்கோட்டை மாட்சிமை தங்கிய மன்னர் கல்லூரி மாணவர்களுக்குப் பரிசுகளும், சான்றிதழ்களும் வழங்கப் பட்டன. சென்னைப் பல்கலைக்கழகத் தமிழ் இலக்கியத்துறைப் பேராசிரியர் முனைவர்.ஆ.குழந்தை வீரமாமுனிவரின் ஆளுமைகள் என்கிற மையப்பொருளில் சிறப்புரை ஆற்றினார். ஆய்வு மாணவர் பா.எழில்செல்வன் நிகழ்வுகளைத் தொகுத்து வழங்கினார். இவ்விழாவை யொட்டி நடத்தப்பட்ட போட்டிகளில் பரிசு பெற்ற வேட்டவலம் லயோலா கல்லூரி, திருச்சி தேசிய கல்லூரி மாணவர்கள் உட்பட, தூய வளனார் கலலூரி தமிழ் மற்றும் வரலாற்றுத் துறை இளங்கலை, முதுகலை மாணவர்கள், பேராசிரியர்கள்,ஆசா அவர்களின் குடும்பத்தினர், தமிழ்ஆர்வலர்கள், வழக்கறிஞர்கள் பங்கேற்றனர். நிறைவில் வளனார் தமிழ்ப் பேரவைப் பொறுப்பாளர் முனைவர்.போ.ஜான்சன் நன்றியுரையாற்றினார்.

போர்முரசு, சென்னை 22.02.2021

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தமிழாய்வுத்துறை
தூய வளனார் கல்லூரி (தன்னாட்சி)
திருச்சிராப்பள்ளி - 2.

ஆசா அறக்கட்டளை நடத்தும்

வீரமாமுனிவர் படைப்புகளில்
கல்லூரி மாணவர்களுக்கான போட்டிகள்

போட்டிகள் நிகழும் நாள் : 09-03-2021

முதல் பரிசு ரூ. 2000
இரண்டாம் பரிசு ரூ. 1500 **மூன்றாம் பரிசு ரூ. 1000**

பேச்சுப்போட்டி :
தலைப்பு : **"தமிழுக்குத் தொண்டு செய்த வீரமாமுனிவர்"** கால அளவு : 5 மணித்துளிகள்.

கவிதைப்போட்டி :
போட்டிக்கான தலைப்பு போட்டி நாள் அன்று வழங்கப்படும் கவிதை அளவு : 50 அடிகள் வரை.

வினாடி வினா :
கேள்விகள் : **வீரமாமுனிவர் படைப்புகளிலிருந்து 25 வினாக்கள்**
Google Class ல் கொடுக்கப்படும். கால அளவு : 30 நிமிடங்கள்.

போட்டி விதிமுறைகள்

- ஒரு கல்லூரியிலிருந்து ஒரு போட்டியில் இருவர் மட்டுமே பங்கேற்க வேண்டும்.
- ஒருவர் ஒரு போட்டியில் மட்டுமே பங்கேற்க வேண்டும்.
- கவிதை எழுத ஒரு மணிநேரம் வழங்கப்படும், எழுதி முடித்து **Google Class** -ல்பதிவேற்றம் செய்யவேண்டும்.
- பதிவுப் படிவம் பூர்த்தி செய்து அனுப்ப இறுதி நாள் 05.03.2021.
- பதிவுப்படிவத்துடன் தங்களது கல்லூரி அடையாள அட்டையை நகல் எடுத்து அனுப்பவும், நேரில் வந்தும் பதிவுப் படிவத்தைக் கொடுக்கலாம்.
- பதிவுப்படிவத்தில் கொடுக்கப்பட்டுள்ள மின்னஞ்சல் முகவரிக்குத் தொடர்பு இணைப்பு உருவாக்கி, அழைப்பு **(Invite)** கொடுக்கப்படும். அதற்கு ஏற்ப **(Accept)** அளித்தால்தான் போட்டிகளில் கலந்து கொள்ள இயலும், அனைத்துப் போட்டிகளும் **Google Class Room** .ல் நடைபெறும்.

பதிவுப்படிவம் அனுப்புவதற்கான முகவரி :

முனைவர் **ஆ. இராஜாத்தி**
உதவிப்பேராசிரியர், தமிழாய்வுத்துறை, தூய வளனார் கல்லூரி, திருச்சி-2.
அலைபேசி எண் : 96263 68272 Mail Id : rajathi_ta1@mail.sjctni.edu

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தூய வளனார் தன்னாட்சிக் கல்லூரி
(A++ தரத்தகுதியும், செயல்திறன் ஆற்றல் வளத் தனித்தகுதியும்,
சிறப்புத் தொன்மைத் தகுதியும் பெற்றது)
திருச்சிராப்பள்ளி – 02.

V.G. சந்தோசம் அறக்கட்டளை, முனைவர் அ.குழந்தைசாமி அறக்கட்டளை

முத்தமிழ் விழா – 2021

வெற்றி பெற்றவர்களின் விவரம்

வ. எண்.	போட்டி		பரிசு பெறுவோர்
1.	கட்டுரைப்போட்டி	1	வீ. மணிகண்டன் இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு அறிஞர் அண்ணா அரசு கலைக் கல்லூரி முசிறி
		2	ரா. கோபிநாத் இளங்கலைத் தமிழ் முதலாம் ஆண்டு தியாகராசர் கலைக்கல்லூரி, மதுரை
		3	செ. நிர்மலா இளங்கலைத் தமிழ் இரண்டாம் ஆண்டு ஸ்ரீமதி இந்திரா காந்தி கல்லூரி, திருச்சி
2.	கவிதைப்போட்டி	1	சௌ. அஸ்வின் இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு தியாகராசர் கல்லூரி, மதுரை -09
		2	வீ.மணிகண்டன் இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு அறிஞர் அண்ணா அரசு கலைக் கல்லூரி, முசிறி
		3	க. பிரவீன் குமார் இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு தேசியக் கல்லூரி, திருச்சிராப்பள்ளி.
		3	மு.சினேகா இளங்கலைத் தமிழிலக்கியம் மூன்றாம் ஆண்டு இதயா மகளிர் கல்லூரி, கும்பகோணம்.
3.	பேச்சுப்போட்டி	1	**செ.பாலுஆனந்த்** இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு தியாகராசர் கல்லூரி, மதுரை
		2	**பா. சுரேகா** இளங்கலைத் தமிழ் முதலாம் ஆண்டு பாத்திமா கல்லூரி, மதுரை
		3	**ச.வாசுதேவன்** இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு திருவள்ளுவர் பல்கலைக்கழக கலை மற்றும் அறிவியல் கல்லூரி, கள்ளக்குறிச்சி.

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4.	பாட்டுப்போட்டி	1	மஞ்சுளா, இளங்கலைத் தமிழிலக்கியம் இரண்டாம் ஆண்டு பாரதிதாசன் பல்கலைக்கழக உறுப்புக் கலை மற்றும் அறிவியல் கல்லூரி, ஸ்ரீரங்கம்
		2	வி. கோனிக் ஸ்ரேவின் ஸ்ரீ இளங்கலைத் தமிழ் இரண்டாம் ஆண்டு தூய வளனார் கல்லூரி (தன்னாட்சி)
		3	லெ. மோனிஷா இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு தூய வளனார் கல்லூரி (தன்னாட்சி)
5.	நிழற்படம் எடுத்தல்	1	பி.மா.வேதா, இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு சீதாலட்சுமி இராமசாமி கல்லூரி, திருச்சி.
		2	செள. அஸ்வின் இளங்கலைத் தமிழ் மூன்றாம் ஆண்டு தியாகராசர் கல்லூரி, மதுரை -09
		3	ச.திருநாவுக்கரசு, இளங்கலைத் தமிழ் முதலாம் ஆண்டு, இலயோலா கல்லூரி, சென்னை.

சுழற்கோப்பை பெறுதல்

முதலிடம் : தமிழ்த்துறை, தியாகராசர் கல்லூரி, மதுரை
முனைவர் மலர்விழி மங்கையர்கரசி
தமிழ்த்துறைத்தலைவர் அலைபேசி : 8870281530

இரண்டாம் இடம் : தமிழ்த்துறை,
அறிஞர் அண்ணா அரசு கலைக்கல்லூரி, முசிறி
முனைவர் த.மஞ்சுளா தேவி
தமிழ்த்துறைத்தலைவர் அலைபேசி : 9787703625

முனைவர் ஞா.பெஸ்கி
தமிழாய்வுத்துறைத் தலைவர்

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தமிழாய்வுத்துறை

தூய வளனார் கல்லூரி (தன்னாட்சி)

திருச்சிராப்பள்ளி – 620 002.

தொலைபேசி : 0431 - 4226401, 2700320, தொலை நகலி : 0431 - 2701501

இணையதளம் : www.sjctni.edu

15.03.2021

ஆசா அறக்கட்டளை

வீரமாமுனிவர் படைப்புகளில் கல்லூரி மாணவர்களுக்கு 2020 – 2021 ஆம் கல்வியாண்டில் நடத்தப்பெற்ற போட்டிகளின் முடிவுகள்

பேச்சுப்போட்டி

முதல் பரிசு	பா. தமிழரசன், தேசியக் கல்லூரி, திருச்சிராப்பள்ளி
இரண்டாம் பரிசு	அ. விஜயலெட்சுமி, தேசியக் கல்லூரி, திருச்சிராப்பள்ளி.
மூன்றாம் பரிசு	த.ஆரோக்கிய ஆலிவர் ராஜா, தூய வளனார் கல்லூரி, திருச்சிராப்பள்ளி.

கவிதைப்போட்டி

முதல் பரிசு	மு.பிடல் காஸ்ட்ரோ, இலயோலா கல்லூரி, சென்னை.
இரண்டாம் பரிசு	இயே. ஆனந்தி, இலயோலா கல்லூரி, வேட்டவலம்.
மூன்றாம் பரிசு	எம்.பி. ஸ்ரீகாமாட்சி, தேசியக் கல்லூரி, திருச்சிராப்பள்ளி.
(இருவருக்கு)	செ.சினேகிதா, இலயோலா கல்லூரி, வேட்டவலம்.

வினாடி வினாப்போட்டி

முதல் பரிசு	ம.நம்பி, மாட்சிமை தங்கிய மன்னர் கல்லூரி, புதுக்கோட்டை.
இரண்டாம் பரிசு	எல்.மோனிஷா, தூய வளனார் கல்லூரி, திருச்சிராப்பள்ளி.
மூன்றாம் பரிசு	மு.கயல்விழி, இலயோலா கல்லூரி, வேட்டவலம்.

குறிப்பு : 18.03.2021 (வியாழன்) அன்று திருச்சிராப்பள்ளி, தூய வளனார் கல்லூரியில் முற்பகல் 11 மணிக்கு நடைபெறும் ஆசா அறக்கட்டளைச் சொற்பொழிவில் பங்கேற்றுப் பரிசினைப் பெற்றுச் செல்லுமாறு அன்புடன் அழைக்கின்றோம்

Dr. G. BESCHI
Associate Professor & Head
Department of Tamil
St. Joseph's College (Autonomous)
Tiruchirappalli-620 002.

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ACCESS 2020 internship.

TO,

MRS, OCTOVIA ANTONY SESSAMMAL,
HEAD OF THE DEPARTMENT,
DEPARTMENT OF BUSINESS ADMINISITRATION,
ST, JOSEPH'COLLEGE (AUTONOMOUS)
TRICHY- 620002

Greetings from The Times of India !!

Trust you and your family are doing well and keeping safe!

Covid 19 has changed the way business is being conducted. As part of the Times Internship program for the benefit of students who are keen to do internship project as part of their academics, we are happy to announce the digital version this year. We are making every effort to keep Interns safe without disrupting and reaching the existing consumer and potential customer during this pandemic.

We are looking for Students for the period : Work from Home Project - 30 to 45 Days

Please find attached the details on the new Digital Internship Program and Job description.

Request you to kindly float the internship program and share the name of the interested students.

Thanks and Regards

Viswanathan.g.v
8883189527

THE TIMES OF INDIA
NO: 12-B/2, WILLIAMES ROAD,
CANTONMENT, TIRICHY-620 001

DEPARTMENT OF BUSINESS ADMINISTRATION
ST.JOSEPH'S COLLEGE (Autonomous)
Re-accredited with A++Grade by NACC & College with Potential for Excellence
TIRUCHIRAPPALLI – 620 002, TAMIL NADU, INDIA.

REPORT ON DIGITAL INTERNSHIP PROGRAMME "ACCESS 2020"

Department of BBA with the linkage of The Times of India, Contonment, Trichy has conducted a digital internship for our BBA students during the period 1st August to 15th September, 2020.

As part of the Times internship programme for the benefit of students who are keen to do internship as part of their academics this programme has been conducted as a work from home project for a period of 30 to 45 days.

A total of nineteen students willingly participated in the digital internship programme. Through this programme the students have been trained to analyse the marketing environment of the Times of India.

The students have learnt the aspects of identify the customer preference especially towards the Times of India and its other products. All the students paralelly updated the market condition to their company guide.

Finally, at the end of the 45 days the students were appreciated and given a certificate of participation from the Reporting Manager and Head of RMD, The Times of India.

Octovia Antony Sessammal

Prof. C. F. OCTOVIA ANTONY SESSAMMAL, MBA.,M.
Head & Assistant Professor
Department of Business Administration
St. Joseph's College (Autonomous)
Tiruchirappalli - 620 002.

S No	Student Name	College Name	Reg. No	Dept	Year
449	WILFRED R K	St.Joseph's College Trichy	18UBU501	BBA	III
451	JOTHIESWARAN S V	St.Joseph's College Trichy	18UBU505	BBA	III
460	ABDUL FASEET KHAN B K	St.Joseph's College Trichy	18UBU523	BBA	III
464	HARIHARAN P	St.Joseph's College Trichy	18UBU532	BBA	III
470	MANIKANDAN A	St.Joseph's College Trichy	18UBU541	BBA	III
472	ANTONY MALPHILO R	St.Joseph's College Trichy	18UBU544	BBA	III
478	PRIYAN J	St.Joseph's College Trichy	18UBU553	BBA	III
479	RAJMOHAN G	St.Joseph's College Trichy	18UBU556	BBA	III
482	KRISHNA DOSS A	St.Joseph's College Trichy	18UBU564	BBA	III
489	SANDHANA YASOR J	St.Joseph's College Trichy	18UBU562	BBA	III
490	KISHANTH S C	St.Joseph's College Trichy	18UBU560	BBA	III
493	ARULRAJA A	St.Joseph's College Trichy	18UBU550	BBA	III
494	GIDIYON J	St.Joseph's College Trichy	18UBU510	BBA	III
501	SURYA KRIS K	St.Joseph's College Trichy	18ubu608	BBA	III
505	MANIKANDAN S	St.Joseph's College Trichy	18ubu612	BBA	III
514	A.Brejesh Paul	St.Joseph's College Trichy	19UBU613	II-BBA 'B'	II
516	J.Infant Ajay	St.Joseph's College Trichy	19UBU672	II-BBA 'B'	II
517	A.Micheal Dalwin	St.Joseph's College Trichy	19UBU661	II-BBA 'B'	II
525	**Hariharan**	St.Joseph's College Trichy	**19ust167**	**BSC statistics**	**II**
526	**Veeramani . V**	St.Joseph's College Trichy	**19ubu540**	**BBA**	**II**

THE TIMES OF INDIA

ACCESS

Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms~~. ...J. INFANT AJAY......... of ......ST. JOSEPH'S COLLEGE [BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT...............

G. V. [signature] .... | [signature] ....

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

Head of RMD,
The Times of India

THE TIMES OF INDIA

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Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ A. MICHEAL DALWIN of ST. JOSEPH'S COLLEGE [BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as EXCELLENT

Reporting Manager

THE TIMES OF INDIA TRICHY - 620001

Head of RMD,
The Times of India

THE TIMES OF INDIA

ACCESS

Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ......A. BREJESH PAUL...... of ....ST. JOSEPH'S COLLEGE [BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT................

G.V. [signature]

Reporting Manager

THE TIMES OF INDIA * TRICHY - 620001 *

[signature]

Head of RMD,
The Times of India

THE TIMES OF INDIA

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This is to certify that

Mr./ ~~Ms.~~ ........SURYA...KRIS...K........ of ....ST...JOSEPH'S.....COLLEGE...[BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT............

....

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

....

Head of RMD,
The Times of India

THE TIMES OF INDIA

ACCESS

Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms~~. ....VEERAMANI . V............ of ....St. JOSEPH'S COLLEGE....[BBA]..

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT...............

G.V. [signature]

....

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

[signature]

....

Head of RMD,
The Times of India

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Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms~~. ....MANIKANDAN S......... of ....St. JOSEPH'S....COLLEGE...[BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....GOOD........................

G.V. [signature] .... Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

[signature] .... Head of RMD, The Times of India

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Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ....KRISHNA DOSS....A........ of ....ST. JOSEPH'S....COLLEGE...[BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT..........

G.V. [signature]

....

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

[signature]

....

Head of RMD,
The Times of India

THE TIMES OF INDIA

ACCESS

Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ......SANDHANA YASAR J. of ..S.T. JOSEPH'S COLLEGE...[B.B.A]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT............

G.V. [signature] ....

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

[signature] ....

Head of RMD,
The Times of India

THE TIMES OF INDIA

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Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ....KISHANTH S.C...... of ....ST. JOSEPH'S COLLEGE [BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT...............

....

Reporting Manager

THE TIMES OF INDIA
★ TRICHY - 620001 ★

....

Head of RMD,
The Times of India

THE TIMES OF INDIA

ACCESS

Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ....RAJMOHAN. G........ of ....ST. JOSEPH'S....COLLEGE..[BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT...........

G.V. [signature]

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

[signature]

Head of RMD,
The Times of India

THE TIMES OF INDIA

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Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ....PRIYAN J............ of ....ST. JOSEPH'S COLLEGE.... [BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT............

....

Reporting Manager

THE TIMES OF INDIA
* TRICHY - 620001 *

....

Head of RMD,
The Times of India

THE TIMES OF INDIA

ACCESS

Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ....ARUL RAJA...A............... of ..S.T...J.O.S.E.P.H'S....C.O.L.L.E.G.E....[B.B.A]..

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....GOOD......................

G.V. [signature] ....

Reporting Manager

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[signature] ....

Head of RMD,
The Times of India

THE TIMES OF INDIA

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Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ANTONY MALPHILO R of ST. JOSEPH'S COLLEGE [BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as EXCELLENT

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

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The Times of India

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ACCESS

Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ......MANIKANDAN...A...... of ..ST...JOSEPH'S...COLLEGE...[BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT............

....

Reporting Manager

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Head of RMD,
The Times of India

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ACCESS

Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ....HARIHARAN P.......... of ..ST. JOSEPH'S COLLEGE [BBA]...

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT..............

....

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

....

Head of RMD,
The Times of India

THE TIMES OF INDIA

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Student's Window to the Corporate World

This is to certify that

Mr/ — ABDUL FASEET KHAN B K of S.T. JOSEPH'S COLLEGE [BBA]

candidate has successfully completed the Times Access internship program of 2020-21.

His/Her performance during the program is adjudged as EXCELLENT

Reporting Manager

THE TIMES OF INDIA * TRICHY - 620001

Head of RMD,
The Times of India

THE TIMES OF INDIA

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Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms~~. ........G.I.D.I.Y.O.N...J........... of .....S.T...J.O.S.E.P.H.'S....COLLEGE [BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT...........

....
Reporting Manager

THE TIMES OF INDIA * TRICHY - 620001 *

....
Head of RMD,
The Times of India

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Student's Window to the Corporate World

This is to certify that

Mr./ ~~Ms.~~ ....JOTHI ESWARAN S V.... of ....ST. JOSEPH'S COLLEGE....[BBA]

institute has successfully completed the Times Access internship program of 2020-21.

His/ her performance during the program is adjudged as ....EXCELLENT..............

G.V. ....

Reporting Manager

THE TIMES OF INDIA ★ TRICHY - 620001 ★

....

Head of RMD,
The Times of India

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Student's Window to the Corporate World

This is to certify that

Mr. — WILFRED R K of ST. JOSEPH'S COLLEGE [BBA]

has successfully completed the Times Access internship program of 2020-21.

performance during the program is adjudged as GOOD

Reporting Manager

THE TIMES OF INDIA TRICHY 620001

Head of RMD,
The Times of India

THE INSTITUTE OF COST ACCOUNTANTS OF INDIA
TIRUCHIRAPPALLI CHAPTER
CMA BHAVAN, 48, Bharathidasan salai, Cantonment, TIRUCHIRAPPALLI - 620 001.
Ph : 0431 - 2461662, Mobile : 8903252873
E-mail : tiruchirappalli@icmai.in www.trichychapter-icmai.in

To

Date:19.02.2021

The Head of the Department,

Department of BBA-Shift II,

St.Joseph's College (Autonomous),

Trichy.

Respected Sir / Madam ,

We have conducted Career Counselling Programme to create awareness about the CMA(ICWA) Course in your Department .We have addressed a cross section of all students as per details below,

S.No	Date	Audience	No. Of Students attended
1	15.02.2021	St.Joseph's College (Shift II BBA -III Year)	110
2	17.02.2021	St.Joseph's College (Shift II BBA -I Year, Via Online)	125

We thankful for having accorded permission to organise the above programmes. We are look forward to your continued support in future.

Yours faithfully,

Chairman / Secretary

SECRETARY
THE INSTITUTE OF COST ACCOUNTANTS OF INDIA
TIRUCHIRAPPALLI CHAPTER
CMA BHAVAN, No.48, BHARATHIDASAN SALAI,
CANTONMENT, TIRUCHIRAPPALLI - 620 001.

DEPARTMENT OF BBA

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

REPORT ON CAREER COUNSELLING PROGRAMME

A "Career Counseling Programme" for all the final year BBA students was held on 15th February, 2021 in KPJ Hall and for the first year BBA students through Google Meet platform on 17th Febraury, 2021 at 02:30 pm.

Dr. Augustine Arockiaraj and Mrs. P. Premakuamri, Assistant Professors, Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli introduced the resource persons to the attendees with a warm welcome note on the respective days.

The resource persons Mr. Natarajan Sethuraman FCMA and Mr. Dharmaraj were from The Institute of Cost Accountants of India, Tiruchirappalli Chapter, Tiruchirappalli. On both the days the resource persons enlightened the participants on the various dimensions of succeeding in the career. They gave a clear explanation on the various scopes available after completing the under graduation. The resource persons created awareness about the CMA (ICWA) course to the students.

Finally the programme ended with a question session by 04:00 PM, with a vote of thanks by Mr. S. Arputharaj, and Dr. P. Bastin Arockia Raj, Assistant Professors, Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli.

Octovia Antony Sessammal

Prof. C. F. OCTOVIA ANTONY SESSAMMAL, MBA.,M.Phil.,NET.
Head & Assistant Professor
Department of Business Administration
St. Joseph's College (Autonomous)
Tiruchirappalli - 620 002.

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

"CAREER COUNSELLING PROGRAMME"

RESOURCE PERSON

Mr. P. Natarajan Sethuraman FCMA and Mr. Dharmaraj

The Institute of Cost Accountants of India

Tiruchirappalli Chapter

Tiruchirappalli

Date: 17/02/2021

Venue: Virtual Mode

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

"CAREER COUNSELLING PROGRAMME"

RESOURCE PERSON

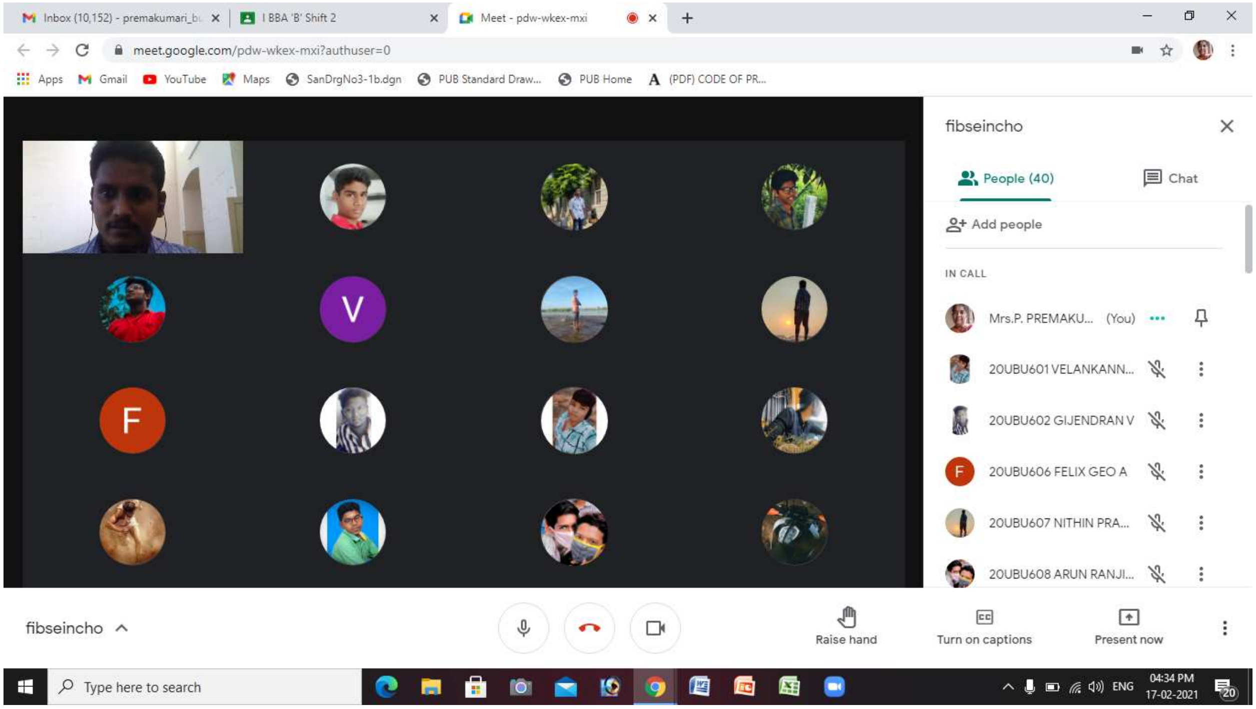
Mr. P Mr. Natarajan Sethuraman FCMA and Mr. Dharmaraj

The Institute of Cost Accountants of India

Tiruchirappalli Chapter

Tiruchirappalli

Date: 15/02/2021 Venue: K.P. Joseph Hall



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I BBA 'B' Shift 2
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YouTube
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20UBU602 GIJENDRAN V
20UBU606 FELIX GEO A
20UBU607 NITHIN PRA...
20UBU608 ARUN RANJI...
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17-02-2021

Profile

Name	NATARAJAN SETHURAMAN FCMA
Qualification	B.Sc (Physics).,M.A.(Tamil, English & Sociology), CMA., & CS
Designation	(Retired) Deputy Manager in Finance
Company	BHEL – Trichy
At Present	Managing Committee Member in "The Institute of Cost Accountants of India", Tiruchirappalli Chapter.
Contact No	9442255037
Email Id	tamilnat@gmail.com

III year Section 'A'

DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002
BATCH: 2018-2021

<u>CAREER COUNSELING PROGRAM</u> (ICAI)

S.NO	REG. NO.	NAME	SIGNATURE
1.	18UBU501	WILFRED. R.K	
2.	18UBU502	CHARLYSARAN. I	
3.	18UBU504	DHARUN. P	
4.	18UBU505	JOTHIESWARAN. S.V	
5.	18UBU506	NITHYANANTHAM. I	
6.	18UBU507	INFANT GEORGE. S	
7.	18UBU509	FREDRICK JOE. S	
8.	18UBU510	GIDIYON. J	
9.	18UBU511	AAKASH	
10.	18UBU512	AROCKIA PRADEEP. J	
11.	18UBU513	SUSIL MELQUE. S	
12.	18UBU514	SANTHOSH SIVAN. N	
13.	18UBU515	STEEWAUGH. A	
14.	18UBU516	REVANTH. J	
15.	18UBU517	SAMSON DANIEL. S	
16.	18UBU519	RAMESH. M	
17.	18UBU520	ARUNKUMAR. V	
18.	18UBU521	KITHIYOUN RAJA. M	
19.	18UBU522	MARIA RUBAN. S	
20.	18UBU523	ABDUL FASEET KHAN. B.K	
21.	18UBU525	ANTONY ARUN. A	
22.	18UBU527	ABILESHWARAN. M	
23.	18UBU528	ALAN ROSHAN. F	
24.	18UBU529	SANKAR. B	
25.	18UBU530	PRAWIN. T	
26.	18UBU532	HARIHARAN. P	
27.	18UBU533	SANTHOSH. M	
28.	18UBU534	ANDRUSH. K	
29.	18UBU535	VENKATESAN. J	
30.	18UBU536	KODIYARASU. S	
31.	18UBU538	SURIYA. J	
32.	18UBU540	PREMKUMAR. V	
33.	18UBU541	MANIKANDAN. A	
34.	18UBU542	SUDHAKARAN. S	
35.	18UBU543	RANJITH KUMAR. J	
36.	18UBU544	ANTONY MALPHILO. R	
37.	18UBU545	GOKUL NATH. R	

III Year Section 'A'

38.	18UBU546	JONE AMALANATHAN. A	
39.	18UBU549	JERSON EBINESH. S	
40.	18UBU550	ARULRAJA. A	
41.	18UBU551	KARAN. S	
42.	18UBU552	VIGNESHWARAN. N	
43.	18UBU553	PRIYAN. J	
44.	18UBU556	RAJMOHAN. G	
45.	18UBU558	MADHAVAN. V	
46.	18UBU559	AJAY. J	
47.	18UBU560	KISHANTH. S.C	
48.	18UBU561	YOGESH. K	
49.	18UBU562	SANDHANA YASOR. J	
50.	18UBU563	SATHYA MURUGAN. S	
51.	18UBU564	KRISHNA DOSS. A	
52.	18UBU565	GOKUL RAJ. R	
53.	18UBU567	ANTONY SAMY. L	
54.	18UBU568	JOHNSON BRUNO. A	
55.	18UBU569	ARUN KUMAR. R	
56.	18UBU570	SYED MOHAMED RAJEK. A.B	
57.	18UBU572	BHARATHIRAJA. S	
58.	18UBU573	KARTHICKRAJA. U	
59.	18UBU574	EMMANUVEL. A	

III Year Section 'B'

DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002
BATCH: 2018-2021

CAREER COUNSELING PROGRAM (I CAI)

S.NO	REG. NO.	NAME	SIGNATURE
1.	18UBU601	AJAY MICHAEL. S	
2.	18UBU602	ROHIT. D	
3.	18UBU603	NITHISH KUMAR. R	
4.	18UBU604	CLARANCE KISHORE. M	
5.	18UBU605	SRIHARISUGAN. B	
6.	18UBU606	ARTHUR WILSON. J	
7.	18UBU607	AKASH JOSE. A	
8.	18UBU608	SURYA KRIS. K	
9.	18UBU609	FRANCIS XAVIER. S	
10.	18UBU610	ANTRO JERIN. A	
11.	18UBU611	SUESON. I	
12.	18UBU612	MANIKANDAN. S	
13.	18UBU613	AROCKIA PRAVEEN. A	
14.	18UBU614	KARANKUMAR. S	
15.	18UBU616	KIRUBAKARAN. G	
16.	18UBU617	AROCKIA NELSON. S	
17.	18UBU618	RUBAN SAJI. L	
18.	18UBU619	MADHESH KUMAR. M	
19.	18UBU621	VARATHARAJAN. C	
20.	18UBU622	ALJIN. V.J	
21.	18UBU623	MAHENDIRAN. S	
22.	18UBU625	RAGHUL KRISHNA. K	
23.	18UBU626	DINAKARAN. A	
24.	18UBU627	VELAN. N	
25.	18UBU628	ANDREW ARUNPRASATH. G	
26.	18UBU629	ASWIN. L	
27.	18UBU630	MUHILAN. K	
28.	18UBU631	SRIDHAR. C	
29.	18UBU632	DEEPAK ALLWIN. A	
30.	18UBU635	MANUEL FERNANDO. J	
31.	18UBU636	ARAVINTH KUMAR. L	
32.	18UBU637	EDISONIGNESH. A	
33.	18UBU639	CUDSON JOHNNI. S	
34.	18UBU640	LOGESH WARAN. V.P	
35.	18UBU641	CHANDRU	
36.	18UBU642	ILAMPIRAI. R	

III year Section 'B'

37.	18UBU643	MUTHAIYA. A	A. Muthaiya
38.	18UBU644	KARTHICK. M	
39.	18UBU645	BHARATHI. P	P. Bharathi
40.	18UBU646	LAKSHMANAN. A	
41.	18UBU647	RAJESHKUMAR. P	
42.	18UBU648	ANSELIN KENSEN. A.M	A.M. Anselin Kensen
43.	18UBU649	SIMSON. C	C. Simson
44.	18UBU650	MUKESH. N	
45.	18UBU652	JACKMARTIN. D	
46.	18UBU653	HARIHARAN. D	
47.	18UBU654	RISHOK STALIN. S	S. [illegible]
48.	18UBU656	STEPHEN. S	S. Stephen
49.	18UBU657	ALLWIN IMMANUVEL. M	M. [illegible]
50.	18UBU662	SUBASH. K	Subash. K
51.	18UBU663	VISWA RAJ. K	
52.	18UBU664	YUVARAJ. T	
53.	18UBU665	KEVIN ANDREW. R	[illegible] R
54.	18UBU666	MAHESHWARAN. S	
55.	18UBU667	RAGU. S	[illegible]
56.	18UBU668	GRESH KUMAR. P	P. [illegible]
57.	18UBU669	HARIHARAN. K	K. [illegible]
58.	18UBU670	RIYAS KHAN. S	
59.	18UBU671	ASATH. J	J. [illegible]
60.	18UBU672	KISHORE KUMAR. N	[illegible]
61.	18UBU674	VASANTH. S	

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

"LEADERSHIP TRAINING PROGRAMME"

RESOURCE PERSON

Prof. S. JOTHIRAMALINGAM

Head – Center for Leadership

Firebird Institute of Research in Management, Coimbatore.

Date: 19/02/2021 Venue: Placement Cell

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

REPORT ON LEADERSHIP TRAINING PROGRAMME

On 19th February, 2021 a "LEADERSHIP TRAINING PROGRAMME" for all the final year BBA students was conducted in the Placement Cell, St. Joseph's College (Autonomous), Triuchirappalli.

The programme started with a prayer song at 02:00 PM. Following the prayer song, Mr. S. Arputharaj, Assistant Professor, Department of Business Administration, St. Joseph's College (Autonomous), Triuchirappalli introduced the resource person to the participants with a warm welcome note.

Prof. S. Jothiramalingam, Head – Center for Leadership, Firebird Institute of Research in Management, Coimbatore was the resource person. He started the training by 02:05 pm. Initially, he segregated the entire strength of the students into different teams. Then, he conducted different team activities for them and made the training session lively.

The students actively participated in all team activates. Through this training programme the resource person explained the importance of leadership qualities and made the students learn how to improve their leadership qualities.

Finally the training programme ended by 04:30 pm, with a vote of thanks by Mr. D. Rinaldo De David, Assistant Professor, Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli.

Octovia Antony Sessammal

Prof. C. F. OCTOVIA ANTONY SESSAMMAL, MBA.,M.Phil.,NET.
Head & Assistant Professor
Department of Business Administration
St. Joseph's College (Autonomous)
Tiruchirappalli - 620 002.

DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002
BATCH: 2018-2021

LEADERSHIP TRAINING PROGRAMME

S.NO	REG. NO.	NAME	SIGNATURE
1.	18UBU501	WILFRED. R.K	
2.	18UBU502	CHARLYSARAN. I	
3.	18UBU504	DHARUN. P	
4.	18UBU505	JOTHIESWARAN. S.V	
5.	18UBU506	NITHYANANTHAM. I	
6.	18UBU507	INFANT GEORGE. S	
7.	18UBU509	FREDRICK JOE. S	
8.	18UBU510	GIDIYON. J	
9.	18UBU511	AAKASH	
10.	18UBU512	AROCKIA PRADEEP. J	
11.	18UBU513	SUSIL MELQUE. S	SUSIL
12.	18UBU514	SANTHOSH SIVAN. N	N. Santhosh
13.	18UBU515	STEEWAUGH. A	
14.	18UBU516	REVANTH. J	REVANTH.
15.	18UBU517	SAMSON DANIEL. S	S. Samson Daniel
16.	18UBU519	RAMESH. M	
17.	18UBU520	ARUNKUMAR. V	
18.	18UBU521	KITHIYOUN RAJA. M	
19.	18UBU522	MARIA RUBAN. S	
20.	18UBU523	ABDUL FASEET KHAN. B.K	
21.	18UBU525	ANTONY ARUN. A	
22.	18UBU527	ABILESHWARAN. M	
23.	18UBU528	ALAN ROSHAN. F	
24.	18UBU529	SANKAR. B	
25.	18UBU530	PRAWIN. T	
26.	18UBU532	HARIHARAN. P	
27.	18UBU533	SANTHOSH. M	
28.	18UBU534	ANDRUSH. K	
29.	18UBU535	VENKATESAN. J	
30.	18UBU536	KODIYARASU. S	
31.	18UBU538	SURIYA. J	
32.	18UBU540	PREMKUMAR. V	
33.	18UBU541	MANIKANDAN. A	
34.	18UBU542	SUDHAKARAN. S	
35.	18UBU543	RANJITH KUMAR. J	
36.	18UBU544	ANTONY MALPHILO. R	
37.	18UBU545	GOKUL NATH. R	

38.	18UBU546	JONE AMALANATHAN. A	
39.	18UBU549	JERSON EBINESH. S	
40.	18UBU550	ARULRAJA. A	A. Arul
41.	18UBU551	KARAN. S	S. Karan
42.	18UBU552	VIGNESHWARAN. N	
43.	18UBU553	PRIYAN. J	J. Priyan
44.	18UBU556	RAJMOHAN. G	
45.	18UBU558	MADHAVAN. V	V. Madhavan
46.	18UBU559	AJAY. J	
47.	18UBU560	KISHANTH. S.C	
48.	18UBU561	YOGESH. K	
49.	18UBU562	SANDHANA YASOR. J	
50.	18UBU563	SATHYA MURUGAN. S	
51.	18UBU564	KRISHNA DOSS. A	
52.	18UBU565	GOKUL RAJ. R	R. Gokul.
53.	18UBU567	ANTONY SAMY. L	
54.	18UBU568	JOHNSON BRUNO. A	
55.	18UBU569	ARUN KUMAR. R	R. Arunkumar
56.	18UBU570	SYED MOHAMED RAJEK. A.B	
57.	18UBU572	BHARATHIRAJA. S	
58.	18UBU573	KARTHICKRAJA. U	
59.	18UBU574	EMMANUVEL. A	

DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002
BATCH: 2018-2021

LEADERSHIP TRAINING PROGRAMME

S.NO	REG. NO.	NAME	SIGNATURE
1.	18UBU601	AJAY MICHAEL. S	S. Ajay
2.	18UBU602	ROHIT. D	D. Rohit
3.	18UBU603	NITHISH KUMAR. R	R. Nithish Kumar
4.	18UBU604	CLARANCE KISHORE. M	
5.	18UBU605	SRIHARISUGAN. B	B. Sriharisugan
6.	18UBU606	ARTHUR WILSON. J	J. Arthur Wilson
7.	18UBU607	AKASH JOSE. A	
8.	18UBU608	SURYA KRIS. K	
9.	18UBU609	FRANCIS XAVIER. S	
10.	18UBU610	ANTRO JERIN. A	
11.	18UBU611	SUESON. I	
12.	18UBU612	MANIKANDAN. S	
13.	18UBU613	AROCKIA PRAVEEN. A	Arockia Praveen
14.	18UBU614	KARANKUMAR. S	
15.	18UBU616	KIRUBAKARAN. G	
16.	18UBU617	AROCKIA NELSON. S	
17.	18UBU618	RUBAN SAJI. L	
18.	18UBU619	MADHESH KUMAR. M	M. Madhesh
19.	18UBU621	VARATHARAJAN. C	
20.	18UBU622	ALJIN. V.J	
21.	18UBU623	MAHENDIRAN. S	S. [illegible]
22.	18UBU625	RAGHUL KRISHNA. K	
23.	18UBU626	DINAKARAN. A	A. Dinakaran
24.	18UBU627	VELAN. N	N. Velan
25.	18UBU628	ANDREW ARUNPRASATH. G	[illegible]
26.	18UBU629	ASWIN. L	
27.	18UBU630	MUHILAN. K	
28.	18UBU631	SRIDHAR. C	
29.	18UBU632	DEEPAK ALLWIN. A	A. Deepak allwin
30.	18UBU635	MANUEL FERNANDO. J	J manuel
31.	18UBU636	ARAVINTH KUMAR. L	
32.	18UBU637	EDISONIGNESH. A	A. Edison
33.	18UBU639	CUDSON JOHNNI. S	
34.	18UBU640	LOGESH WARAN. V.P	
35.	18UBU641	CHANDRU . R	R. Ch
36.	18UBU642	ILAMPIRAI. R	

37.	18UBU643	MUTHAIYA. A	
38.	18UBU644	KARTHICK. M	
39.	18UBU645	BHARATHI. P	P. Bharathi.
40.	18UBU646	LAKSHMANAN. A	
41.	18UBU647	RAJESHKUMAR. P	
42.	18UBU648	ANSELIN KENSEN. A.M	A.M. Anselinkensen
43.	18UBU649	SIMSON. C	
44.	18UBU650	MUKESH. N	N. Mukesh.
45.	18UBU652	JACKMARTIN. D	
46.	18UBU653	HARIHARAN. D	
47.	18UBU654	RISHOK STALIN. S	
48.	18UBU656	STEPHEN. S	
49.	18UBU657	ALLWIN IMMANUVEL. M	
50.	18UBU662	SUBASH. K	Subash. K
51.	18UBU663	VISWA RAJ. K	
52.	18UBU664	YUVARAJ. T	
53.	18UBU665	KEVIN ANDREW. R	
54.	18UBU666	MAHESHWARAN. S	
55.	18UBU667	RAGU. S	
56.	18UBU668	GRESH KUMAR. P	
57.	18UBU669	HARIHARAN. K	
58.	18UBU670	RIYAS KHAN. S	
59.	18UBU671	ASATH. J	
60.	18UBU672	KISHORE KUMAR. N	
61.	18UBU674	VASANTH. S	

MASTeR

taking wings to new frontiers

Group of Institutions

Siruganur, Tiruchirappalli

St. Joseph's College,
Tiruchirappalli

The Management and Faculty of Department of Business Administration cordially invite you for

CAREER GUIDANCE PROGRAMME

Venue : KPJ Hall.

On Friday 26th February 2021 at 2.00 pm– 3.30 pm

Rev Dr. M. Arockiasamy Xavier SJ
Principal
St. Joseph's College
(Autonomous),
Trichy

Dr. V. Alex Ramani
Deputy Principal
St. Joseph's College
(Autonomous),
Trichy

Ms.C. F. Octovia Antony Sessammal
Head Department of
Business Administration
St. Joseph's College
(Autonomous),
Trichy

Dr. M. Hemalatha
Director
MASTeR Group of
Institution
Trichy

Soliciting your August presence

CAREER GUIDANCE PROGRAMME

Time	Guest Speaker	Session Title
2.00 pm– 3.30 pm	Dr. R. Jimmy Carter Career Development Officer - Master GI Ms. J. Arokiya Monica Placement Officer - MASTeR GI	Career Guidance

DEPARTMENT OF BBA

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

REPORT ON CAREER GUIDANCE PROGRAMME

On 26th February, 2021 a "Career Guidance Programme" for our final year students was held on KPJ Hall. MASTeR Group of Institutions, Siruganur, Tiruchirappalli and Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli jointly organized the programme.

Prof. R. Jimmy Carter, Career Development Officer and Ms. J. Arokiya Monica, Placement Officer – MASTeR Group of Institutions, Siruganur, Tiruchirappalli were the resource persons for the programme. The programme strated at 02:00 pm, Prof. C. F. Octovia Antony Sessammal, HOD, Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli welcomed the resource persons with her welcome address. Immediately after that at 02:05 pm the session was handed over to the resource person.

The programme was divided into two sessions. Session one was handled by Prof. R. Jimmy Carter. He clearly explained the importance and the various other aspects of the entrepreneurship in detail. Also he explained the various funding sources for entrepreneur to our students.

The second session of the programme was handled by Ms. J. Arokiya Monica. She explained about the different sources and means for various job opportunities available for the students in detail. By 04:00 pm the progaramme was concluded with the vote of thanks give by Prof. Augustine Arockiaraj, Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli.

Octovia Antony Sessammal

C. F. OCTOVIA ANTONY SESSAMMAL
Head & Assistant Professor
Department of Business Administration
St. Joseph's College (Autonomous)
Tiruchirappalli - 620 002.

DEPARTMENT OF BBA

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

CAREER GUIDANCE PROGRAMME ON 26TH FEBRUARY, 2021


All progress
takes place outside
the comfort zone.
MICHAEL JOHN BOBAK

DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002
BATCH: 2018-2021
CAREER GUIDANCE PROGRAM

S.NO	REG. NO.	NAME	ATTENDANCE
1.	18UBU501	WILFRED. R.K	
2.	18UBU502	CHARLYSARAN. I	
3.	18UBU504	DHARUN. P	
4.	18UBU505	JOTHIESWARAN. S.V	[illegible]
5.	18UBU506	NITHYANANTHAM. I	
6.	18UBU507	INFANT GEORGE. S	[illegible]
7.	18UBU509	FREDRICK JOE. S	[illegible]
8.	18UBU510	GIDIYON. J	
9.	18UBU511	AAKASH	
10.	18UBU512	AROCKIA PRADEEP. J	[illegible]
11.	18UBU513	SUSIL MELQUE. S	
12.	18UBU514	SANTHOSH SIVAN. N	N. Santhosh
13.	18UBU515	STEEWAUGH. A	[illegible]
14.	18UBU516	REVANTH. J	
15.	18UBU517	SAMSON DANIEL. S	S - Samson Daniel
16.	18UBU519	RAMESH. M	
17.	18UBU520	ARUNKUMAR. V	
18.	18UBU521	KITHIYOUN RAJA. M	[illegible]
19.	18UBU522	MARIA RUBAN. S	
20.	18UBU523	ABDUL FASEET KHAN. B.K	
21.	18UBU525	ANTONY ARUN. A	
22.	18UBU527	ABILESHWARAN. M	
23.	18UBU528	ALAN ROSHAN. F	
24.	18UBU529	SANKAR. B	
25.	18UBU530	PRAWIN. T	
26.	18UBU532	HARIHARAN. P	
27.	18UBU533	SANTHOSH. M	
28.	18UBU534	ANDRUSH. K	K. Andrush
29.	18UBU535	VENKATESAN. J	[illegible]
30.	18UBU536	KODIYARASU. S	
31.	18UBU538	SURIYA. J	
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35.	18UBU543	RANJITH KUMAR. J	
36.	18UBU544	ANTONY MALPHILO. R	[illegible]
37.	18UBU545	GOKUL NATH. R	
38.	18UBU546	JONE AMALANATHAN. A	[illegible]
39.	18UBU549	JERSON EBINESH. S	

40.	18UBU550	ARULRAJA. A	A. Arul
41.	18UBU551	KARAN. S	
42.	18UBU552	VIGNESHWARAN. N	N. Vignesh [illegible]
43.	18UBU553	PRIYAN. J	J. Priyan
44.	18UBU556	RAJMOHAN. G	
45.	18UBU558	MADHAVAN. V	V. Madhavan
46.	18UBU559	AJAY. J	
47.	18UBU560	KISHANTH. S.C	
48.	18UBU561	YOGESH. K	[illegible]
49.	18UBU562	SANDHANA YASOR. J	J. Santhanayasar
50.	18UBU563	SATHYA MURUGAN. S	S. Sathyam
51.	18UBU564	KRISHNA DOSS. A	
52.	18UBU565	GOKUL RAJ. R	
53.	18UBU567	ANTONY SAMY. L	L. Antony Samy
54.	18UBU568	JOHNSON BRUNO. A	
55.	18UBU569	ARUN KUMAR. R	R. Arunkumar
56.	18UBU570	SYED MOHAMED RAJEK. A.B	
57.	18UBU572	BHARATHIRAJA. S	
58.	18UBU573	KARTHICKRAJA. U	
59.	18UBU574	EMMANUVEL. A	

DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002
BATCH: 2018-2021

CAREER GUIDANCE PROGRAM

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5.	18UBU605	SRIHARISUGAN. B	
6.	18UBU606	ARTHUR WILSON. J	
7.	18UBU607	AKASH JOSE. A	
8.	18UBU608	SURYA KRIS. K	[illegible]
9.	18UBU609	FRANCIS XAVIER. S	[illegible]
10.	18UBU610	ANTRO JERIN. A	
11.	18UBU611	SUESON. I	[illegible]
12.	18UBU612	MANIKANDAN. S	S. Manikandan.
13.	18UBU613	AROCKIA PRAVEEN. A	
14.	18UBU614	KARANKUMAR. S	
15.	18UBU616	KIRUBAKARAN. G	
16.	18UBU617	AROCKIA NELSON. S	
17.	18UBU618	RUBAN SAJI. L	
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19.	18UBU621	VARATHARAJAN. C	
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21.	18UBU623	MAHENDIRAN. S	
22.	18UBU625	RAGHUL KRISHNA. K	
23.	18UBU626	DINAKARAN. A	A. [illegible]
24.	18UBU627	VELAN. N	
25.	18UBU628	ANDREW ARUNPRASATH. G	
26.	18UBU629	ASWIN. L	
27.	18UBU630	MUHILAN. K	
28.	18UBU631	SRIDHAR. C	
29.	18UBU632	DEEPAK ALLWIN. A	
30.	18UBU635	MANUEL FERNANDO. J	Emanuel.
31.	18UBU636	ARAVINTH KUMAR. L	
32.	18UBU637	EDISONIGNESH. A	
33.	18UBU639	CUDSON JOHNNI. S	S. Cudson Johni
34.	18UBU640	LOGESH WARAN. V.P	
35.	18UBU641	CHANDRU	
36.	18UBU642	ILAMPIRAI. R	
37.	18UBU643	MUTHAIYA. A	
38.	18UBU644	KARTHICK. M	

39.	18UBU645	BHARATHI. P	
40.	18UBU646	LAKSHMANAN. A	A. Lakshman
41.	18UBU647	RAJESHKUMAR. P	
42.	18UBU648	ANSELIN KENSEN. A.M	
43.	18UBU649	SIMSON. C	C. Simpson
44.	18UBU650	MUKESH. N	
45.	18UBU652	JACKMARTIN. D	
46.	18UBU653	HARIHARAN. D	
47.	18UBU654	RISHOK STALIN. S	
48.	18UBU656	STEPHEN. S	
49.	18UBU657	ALLWIN IMMANUVEL. M	[illegible]
50.	18UBU662	SUBASH. K	Subash.K
51.	18UBU663	VISWA RAJ. K	[illegible]
52.	18UBU664	YUVARAJ. T	
53.	18UBU665	KEVIN ANDREW. R	
54.	18UBU666	MAHESHWARAN. S	
55.	18UBU667	RAGU. S	[illegible]
56.	18UBU668	GRESH KUMAR. P	P. [illegible]
57.	18UBU669	HARIHARAN. K	K. [illegible]
58.	18UBU670	RIYAS KHAN. S	
59.	18UBU671	ASATH. J	J. [illegible]
60.	18UBU672	KISHORE KUMAR. N	[illegible]
61.	18UBU674	VASANTH. S	

ST. JOSEPH'S COLLEGE (AUTONOMOUS)

Special Heritage Status Awarded by UGC
Nationally Re-Accredited with A++ (4th Cycle) Grade
College with potential for Excellence
DBT-STAR & DST-FIST sponsored college
TIRUCHIRAPPALLI - 620002

DEPARTMENT OF BUSINESS ADMINISTRATION

Organises
Hands-on Training in

"Stock Trading"

Resource Persons

Mr. R. Venkatesh
Head
CARE Business School, Trichy

Mr. S. Sunil Allan
Skill Trainer
CARE Business School, Trichy

Felicitation

Rev. Dr. S. Peter SJ, Secretary
Rev. Dr. M. Arockiasamy Xavier SJ, Principal
Dr. V. Alex Ramani, Deputy Principal

Venue: TV Hall

Time & Date
12.03.2021 (Friday)
1:00 PM - 4:00 PM

ALL ARE INVITED

MS. C. F. OCTOVIA ANTONOY SESSAMMAL
Head of the Department

MR. D. RINALDO DE DAVID
Association President

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

REPORT ON HANDS-ON TRAINING ON PRACTICAL STOCK TRADING

On 12th March, 2021 a "HANDS-ON TRAINING ON PRACTICAL STOCK TRADING" for all the final year BBA students was conducted in the TV Hall, St. Joseph's College (Autonomous), Triuchirappalli.

The programme started with a prayer song at 02:00 PM. Following the prayer song, Prof. C. F. Octovia Antony Sessammal, HOD, Department of Business Administration, St. Joseph's College (Autonomous), Triuchirappalli felicitated the gathering.

Mr. R. Venkatesh, Head, CARE Business School, Trichy and Mr. S. Sunil Allan, Skill Trainer CARE Business School, Trichy were the resource persons for the programme. They started the training by 02:10 pm. They practically explained the stock trading with the help of relevant website online to our students.

The students actively participated in the training. Through this training programme the resource persons explained the importance of stock trading, how to buy or sell a share and how to make proper investment decisions on shares.

Finally the training programme ended by 04:00 pm, with a vote of thanks by Dr. B. Augustine Arockiaraj, Assistant Professor, Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli.

OctoviaAnty Sessamy

Prof. C. F. OCTOVIA ANTONY SESSAMMAL, MBA.,M.Phil.,NET.
Head & Assistant Professor
Department of Business Administration
St. Joseph's College (Autonomous)
Tiruchirappalli - 620 002.

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002 HANDS-ON TRAINING ON PRACTICAL STOCK TRADING

ON 12TH MARCH, 2021

DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002
BATCH: 2018-2021

HANDS-ON TRAINING ON STOCK TRADING

S.NO	REG. NO.	NAME	SIGNATURE
1.	18UBU501	WILFRED. R.K	
2.	18UBU502	CHARLYSARAN. I	
3.	18UBU504	DHARUN. P	
4.	18UBU505	JOTHIESWARAN. S.V	
5.	18UBU506	NITHYANANTHAM. I	
6.	18UBU507	INFANT GEORGE. S	
7.	18UBU509	FREDRICK JOE. S	
8.	18UBU510	GIDIYON. J	
9.	18UBU511	AAKASH	
10.	18UBU512	AROCKIA PRADEEP. J	
11.	18UBU513	SUSIL MELQUE. S	
12.	18UBU514	SANTHOSH SIVAN. N	
13.	18UBU515	STEEWAUGH. A	
14.	18UBU516	REVANTH. J	
15.	18UBU517	SAMSON DANIEL. S	
16.	18UBU519	RAMESH. M	
17.	18UBU520	ARUNKUMAR. V	
18.	18UBU521	KITHIYOUN RAJA. M	
19.	18UBU522	MARIA RUBAN. S	LEFT
20.	18UBU523	ABDUL FASEET KHAN. B.K	
21.	18UBU525	ANTONY ARUN. A	
22.	18UBU527	ABILESHWARAN. M	
23.	18UBU528	ALAN ROSHAN. F	
24.	18UBU529	SANKAR. B	
25.	18UBU530	PRAWIN. T	
26.	18UBU532	HARIHARAN. P	
27.	18UBU533	SANTHOSH. M	
28.	18UBU534	ANDRUSH. K	
29.	18UBU535	VENKATESAN. J	
30.	18UBU536	KODIYARASU. S	
31.	18UBU538	SURIYA. J	
32.	18UBU540	PREMKUMAR. V	
33.	18UBU541	MANIKANDAN. A	
34.	18UBU542	SUDHAKARAN. S	
35.	18UBU543	RANJITH KUMAR. J	
36.	18UBU544	ANTONY MALPHILO. R	
37.	18UBU545	GOKUL NATH. R	

38.	18UBU546	JONE AMALANATHAN. A	
39.	18UBU549	JERSON EBINESH. S	
40.	18UBU550	ARULRAJA. A	
41.	18UBU551	KARAN. S	
42.	18UBU552	VIGNESHWARAN. N	
43.	18UBU553	PRIYAN. J	
44.	18UBU556	RAJMOHAN. G	
45.	18UBU558	MADHAVAN. V	
46.	18UBU559	AJAY. J	
47.	18UBU560	KISHANTH. S.C	
48.	18UBU561	YOGESH. K	
49.	18UBU562	SANDHANA YASOR. J	
50.	18UBU563	SATHYA MURUGAN. S	
51.	18UBU564	KRISHNA DOSS. A	
52.	18UBU565	GOKUL RAJ. R	
53.	18UBU567	ANTONY SAMY. L	
54.	18UBU568	JOHNSON BRUNO. A	
55.	18UBU569	ARUN KUMAR. R	
56.	18UBU570	SYED MOHAMED RAJEK. A.B	
57.	18UBU572	BHARATHIRAJA. S	
58.	18UBU573	KARTHICKRAJA. U	
59.	18UBU574	EMMANUVEL. A	

DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002
BATCH: 2018-2021

HANDS-ON TRAINING ON STOCK TRADING

S.NO	REG. NO.	NAME	SIGNATURE
1.	18UBU601	AJAY MICHAEL. S	Ajaymic. A
2.	18UBU602	ROHIT. D	D. Rohit
3.	18UBU603	NITHISH KUMAR. R	R. Nithishkumar
4.	18UBU604	CLARANCE KISHORE. M	
5.	18UBU605	SRIHARISUGAN. B	
6.	18UBU606	ARTHUR WILSON. J	
7.	18UBU607	AKASH JOSE. A	
8.	18UBU608	SURYA KRIS. K	K. Surya Kris
9.	18UBU609	FRANCIS XAVIER. S	
10.	18UBU610	ANTRO JERIN. A	
11.	18UBU611	SUESON. I	
12.	18UBU612	MANIKANDAN. S	Manikan
13.	18UBU613	AROCKIA PRAVEEN. A	Arockia Praveen A.
14.	18UBU614	KARANKUMAR. S	
15.	18UBU616	KIRUBAKARAN. G	
16.	18UBU617	AROCKIA NELSON. S	
17.	18UBU618	RUBAN SAJI. L	
18.	18UBU619	MADHESH KUMAR. M	
19.	18UBU621	VARATHARAJAN. C	
20.	18UBU622	ALJIN. V.J	
21.	18UBU623	MAHENDIRAN. S	
22.	18UBU625	RAGHUL KRISHNA. K	
23.	18UBU626	DINAKARAN. A	A. Dinakaran
24.	18UBU627	VELAN. N	N. Velan
25.	18UBU628	ANDREW ARUNPRASATH. G	
26.	18UBU629	ASWIN. L	
27.	18UBU630	MUHILAN. K	
28.	18UBU631	SRIDHAR. C	
29.	18UBU632	DEEPAK ALLWIN. A	
30.	18UBU635	MANUEL FERNANDO. J	J manuel.
31.	18UBU636	ARAVINTH KUMAR. L	
32.	18UBU637	EDISONIGNESH. A	A. Edison
33.	18UBU639	CUDSON JOHNNI. S	
34.	18UBU640	LOGESH WARAN. V.P	
35.	18UBU641	CHANDRU	
36.	18UBU642	ILAMPIRAI. R	R. [illegible]

37.	18UBU643	MUTHAIYA. A	A. Muthaiya
38.	18UBU644	KARTHICK. M	
39.	18UBU645	BHARATHI. P	
40.	18UBU646	LAKSHMANAN. A	
41.	18UBU647	RAJESHKUMAR. P	
42.	18UBU648	ANSELIN KENSEN. A.M	
43.	18UBU649	SIMSON. C	
44.	18UBU650	MUKESH. N	N. Mukesh
45.	18UBU652	JACKMARTIN. D	
46.	18UBU653	HARIHARAN. D	
47.	18UBU654	RISHOK STALIN. S	
48.	18UBU656	STEPHEN. S	
49.	18UBU657	ALLWIN IMMANUVEL. M	
50.	18UBU662	SUBASH. K	Subash. K
51.	18UBU663	VISWA RAJ. K	
52.	18UBU664	YUVARAJ. T	
53.	18UBU665	KEVIN ANDREW. R	
54.	18UBU666	MAHESHWARAN. S	
55.	18UBU667	RAGU. S	
56.	18UBU668	GRESH KUMAR. P	
57.	18UBU669	HARIHARAN. K	
58.	18UBU670	RIYAS KHAN. S	
59.	18UBU671	ASATH. J	
60.	18UBU672	KISHORE KUMAR. N	
61.	18UBU674	VASANTH. S	

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

"AWARENESS PROGRAMME ON DIGITAL MARKETING"

RESOURCE PERSON

Mr. Manoj Jeevagan

CEO

Inimart Digi Solutions, Trichy.

Date: 26/03/2021

Venue: Online

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

REPORT ON AWARENESS PROGRAMME ON "DIGITAL MARKETING"

On 26th March, 2021 an awareness programme on "Digital Marketing" was conducted for all the final year BBA students through online mode.

The programme started with a prayer song at 02:00 PM. Following the prayer song, Dr. B. Augustine Arockiaraj, Assistant Professor, Department of Business Administration, St. Joseph's College (Autonomous), Triuchirappalli introduced the resource person to the participants with a warm welcome note.

Mr. Manoj Jeevagan, CEO, Inimart Digi Solutions, Trichy was the resource person. He started the programme by 02:05 pm. Initially, he explained the basic concepts and various forms of digital marketing. Later, he discussed the scope and opportunities of digital marketing.

The students actively participated in the programme by clarifying doubts then and there. Through this programme the resource person explained the importance of digital marketing and made the students learn how to improve their digital marketing skills.

Finally the programme ended by 04:30 pm, with a vote of thanks by Mr. J. Inigo Papu Vinodhan, Assistant Professor, Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli.

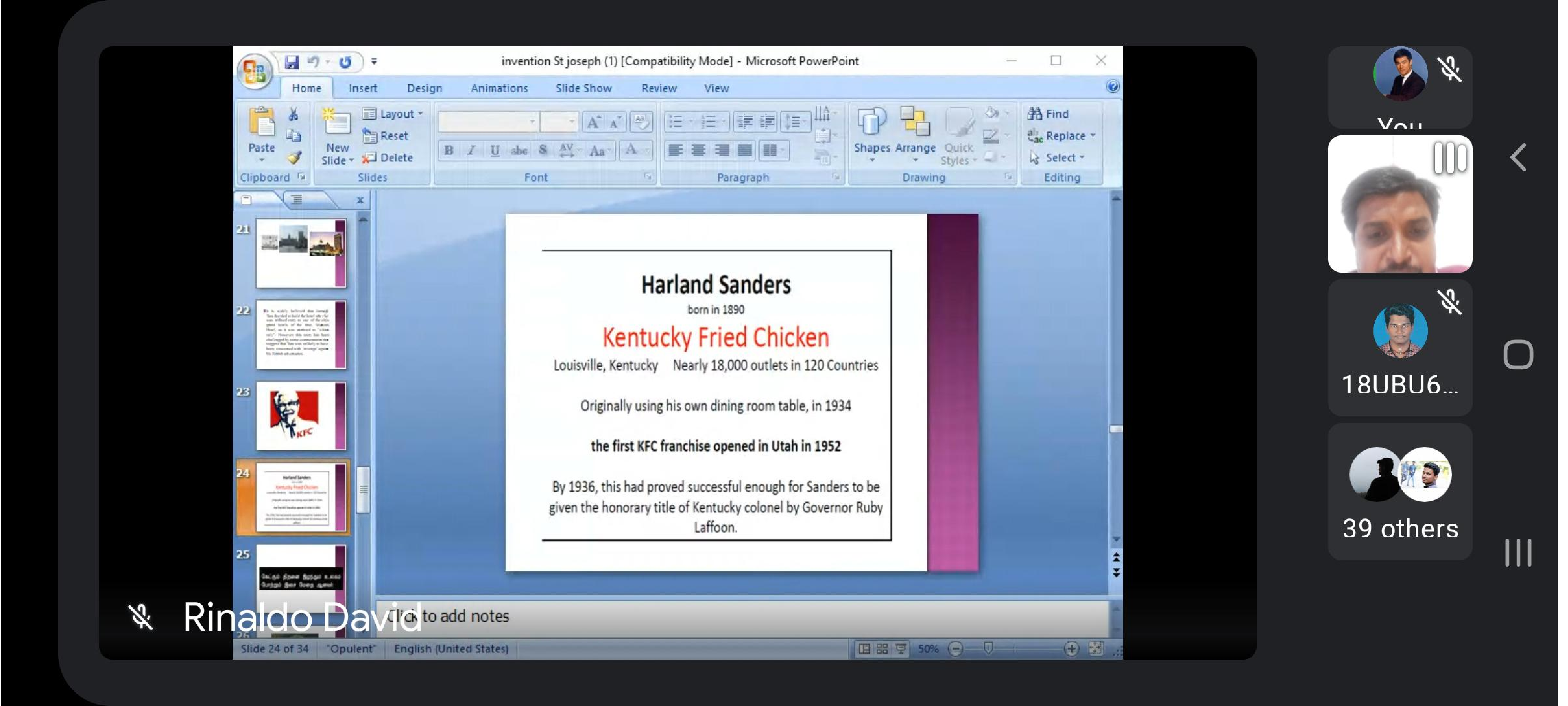
Octovia Antony Sessammal

Prof. C. F. OCTOVIA ANTONY SESSAMMAL, MBA.,M.Phil.,NET
Head & Assistant Professor
Department of Business Administration
St. Joseph's College (Autonomous)
Tiruchirappalli 620 002.

invention St joseph (1) [Compatibility Mode] - Microsoft PowerPoint

Diners Club card was the first national credit card of the United States that was invented by Frank McNamara in 1950 to pay restaurant bills. In 1958, American Express issued their first credit card.

19UBU649 DENISTEN D left


invention St joseph (1) [Compatibility Mode] - Microsoft PowerPoint
Home
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Review
View
Harland Sanders
born in 1890
Kentucky Fried Chicken
Louisville, Kentucky
Nearly 18,000 outlets in 120 Countries
Originally using his own dining room table, in 1934
the first KFC franchise opened in Utah in 1952
By 1936, this had proved successful enough for Sanders to be given the honorary title of Kentucky colonel by Governor Ruby Laffoon.
Click to add notes
Slide 24 of 34
"Opulent"
English (United States)
Rinaldo David
18UBU6...
39 others

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

"INVENTION AND INNOVATION"

RESOURCE PERSON

Prof. R. SHANMUGANATHAN

Assistant Professor

Vijay Institute of Management, Dindigul.

Date: 30/03/2021 Venue: Online

DEPARTMENT OF BUSINESS ADMINISTRATION

ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

REPORT ON SPECIAL LECTURE ON "INVENTION AND INNOVATION"

On 30th March, 2021 a special lecture on "INVENTION AND INNOVATION" for all the final year BBA students was conducted online.

The programme started with a prayer song at 04:00 PM. Following the prayer song, Mr. S. Arputharaj, Assistant Professor, Department of Business Administration, St. Joseph's College (Autonomous), Triuchirappalli introduced the resource person to the participants with a warm welcome note.

Prof. R. Shanmuganathan, Assistant Professor, Vijay Institute of Management, Dindigul was the resource person. He started the programme by 04:05 pm. Initially, he explained about the basic concepts related to inventions and innovations. Later he discussed the concepts by quoting several real life examples.

The students actively participated in the programme. Through this programme the resource person explained the importance of invention and innovation and made the students learn how to improve their innovative qualities.

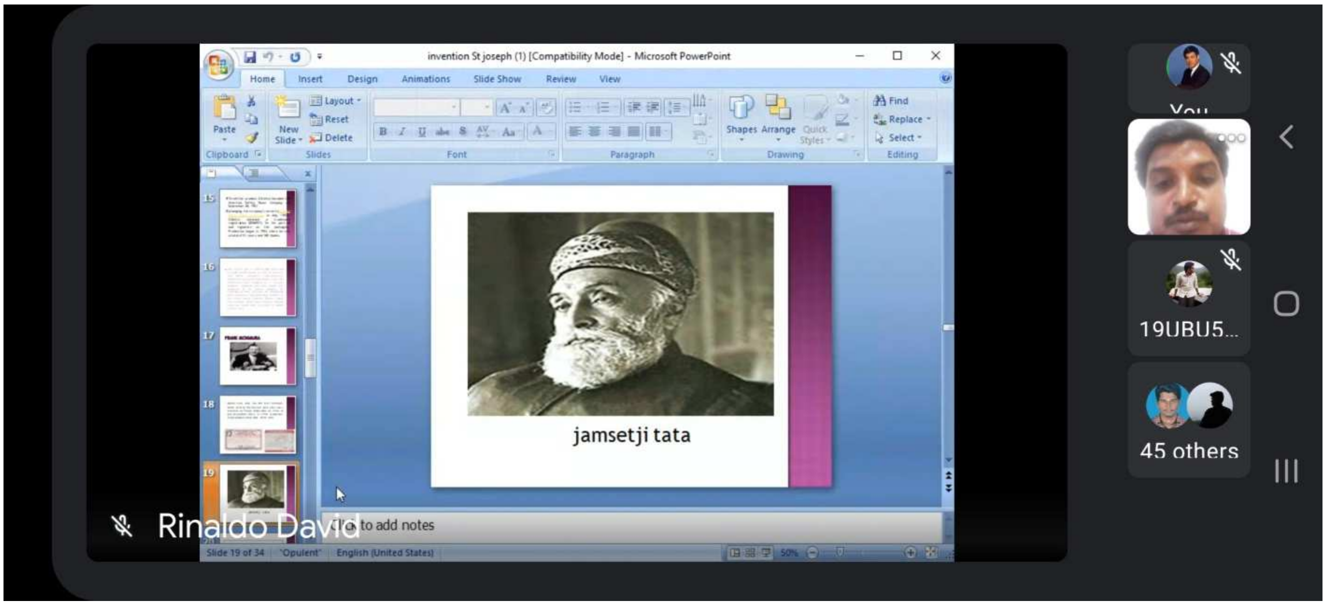
Finally the programme ended by 05:15 pm, with a vote of thanks by Mr. D. Rinaldo De David, Assistant Professor, Department of Business Administration, St. Joseph's College (Autonomous), Tiruchirappalli.

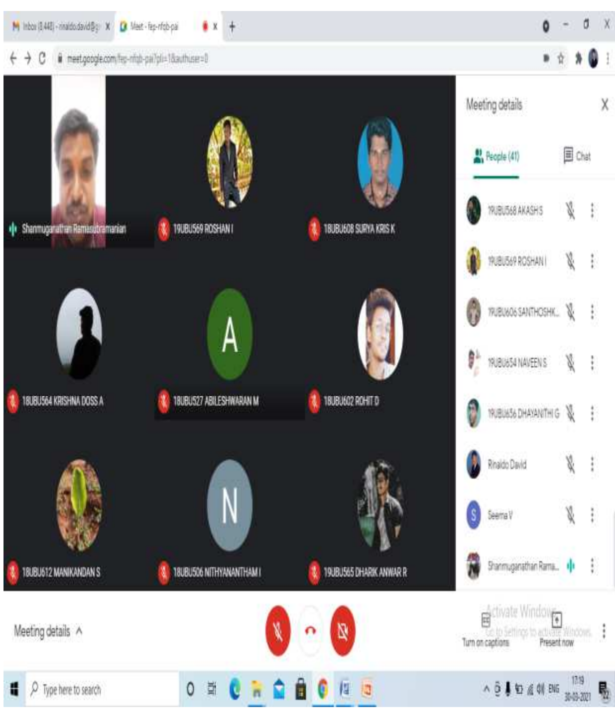
Octovia Antony Sessammal

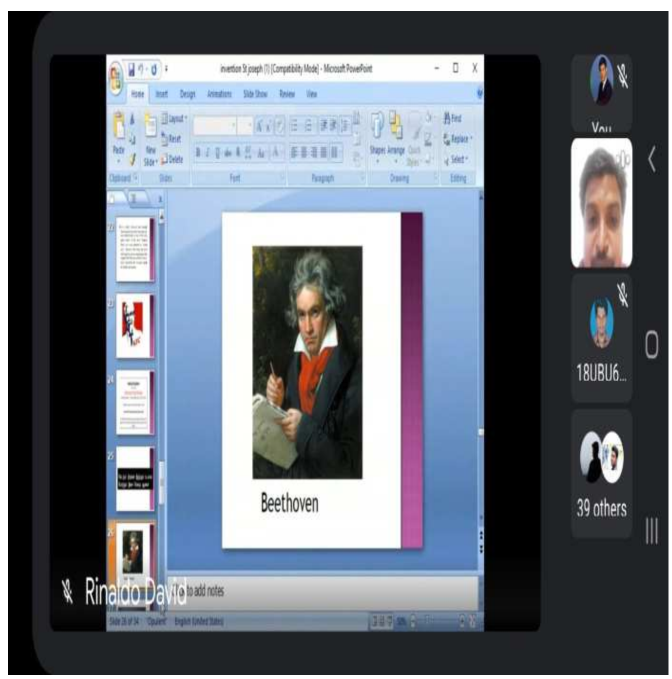
C. F. OCTOVIA ANTONY SESSAMMAL, MBA.,M.Phil.,NET.,
Head & Assistant Professor
Department of Business Administration
St. Joseph's College (Autonomous)
Tiruchirappalli - 620 002.

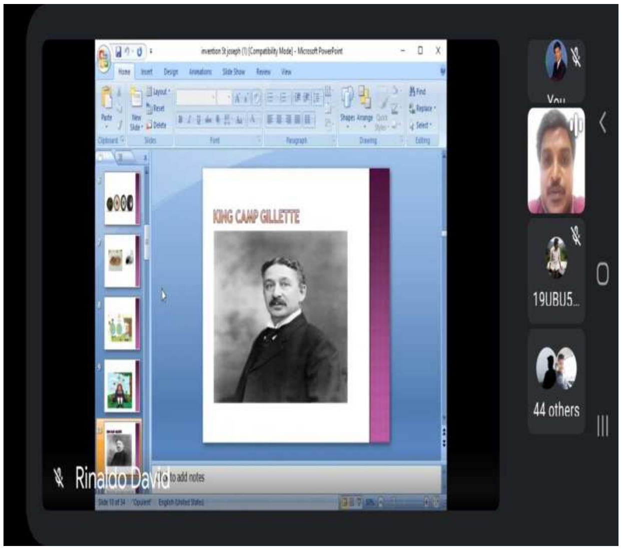
DEPARTMENT OF BUSINESS ADMINISTRATION
ST. JOSEPH'S COLLEGE (AUTONOMOUS), TIRUCHIRAPPALLI – 620 002

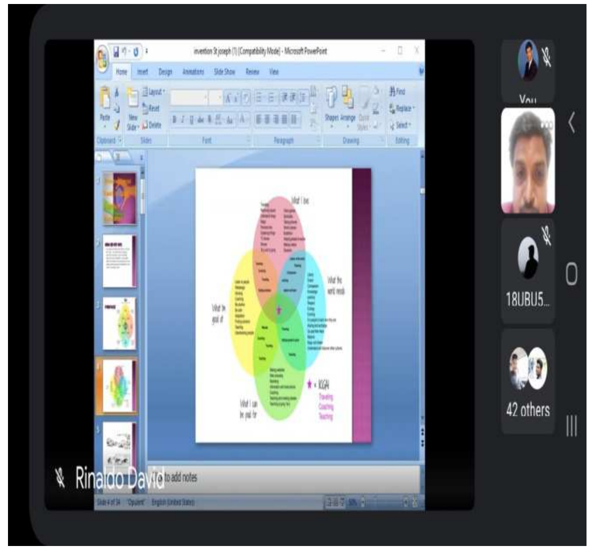
A SPECIAL LECTURE ON"INVENTION AND INNOVATION"
ON 30TH MARCH, 2021
FOR ALL THE FINAL YEAR BBA STUDENTS WAS CONDUCTED ONLINE.

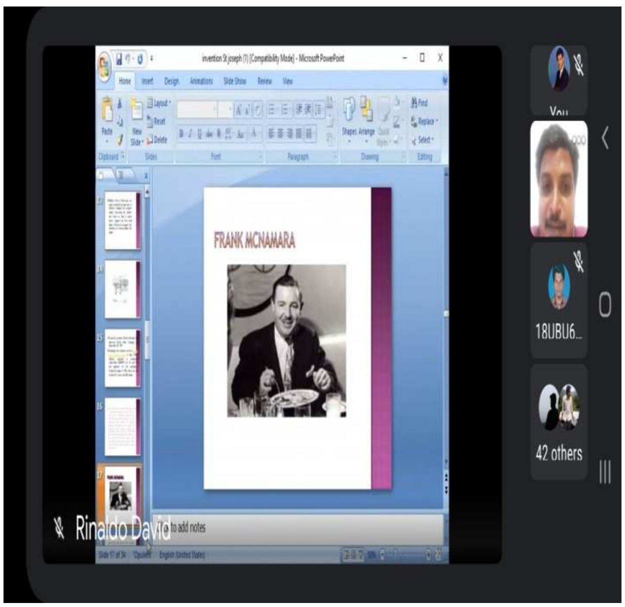


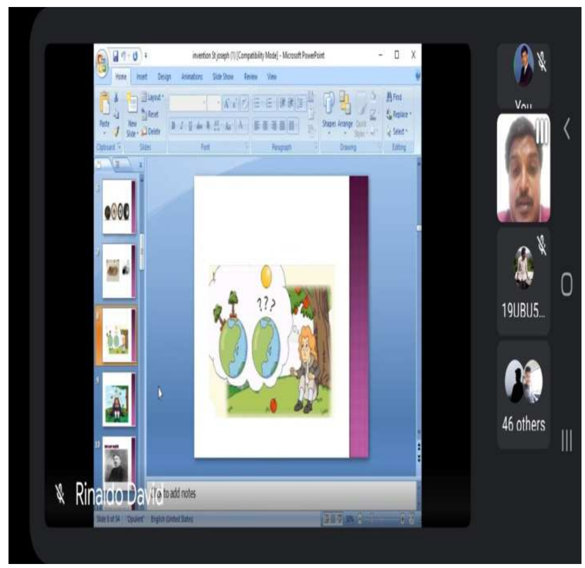


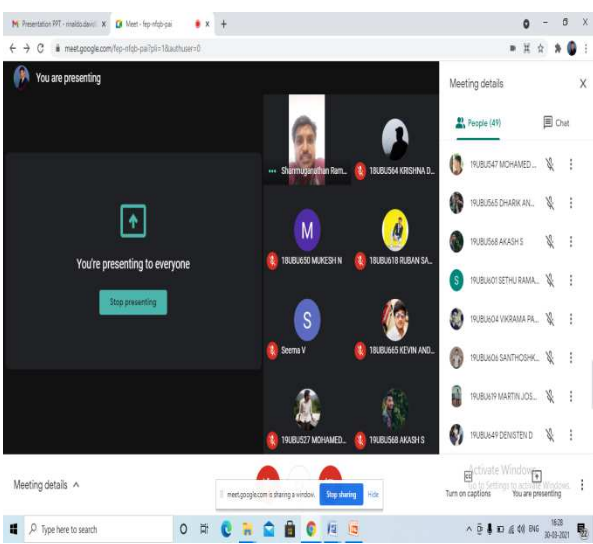


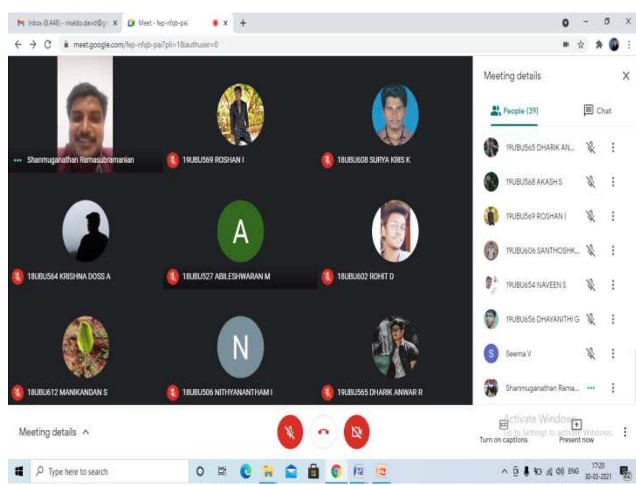

KING CAMP GILLETTE
Rinaldo David
19UBU5...
44 others

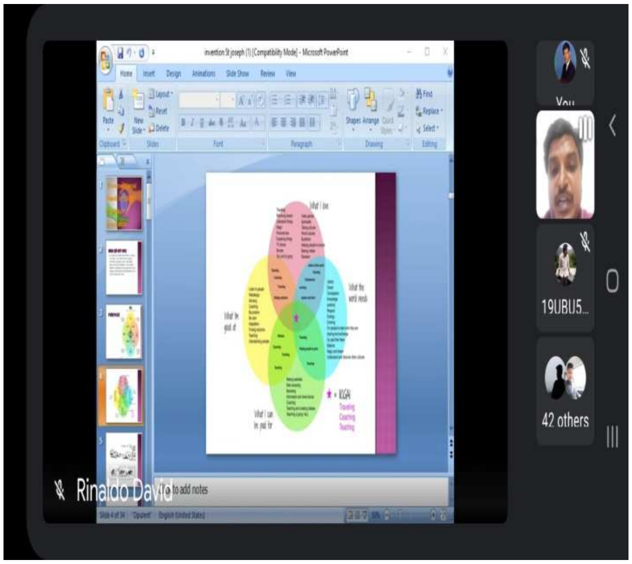

Rinaldo David
18UBU5...
42 others


FRANK MCNAMARA
Rinaldo David
18UBU6...
42 others


Rinaldo David
19UBU5...
46 others


You are presenting
You're presenting to everyone
Stop presenting
Meeting details


Meeting details



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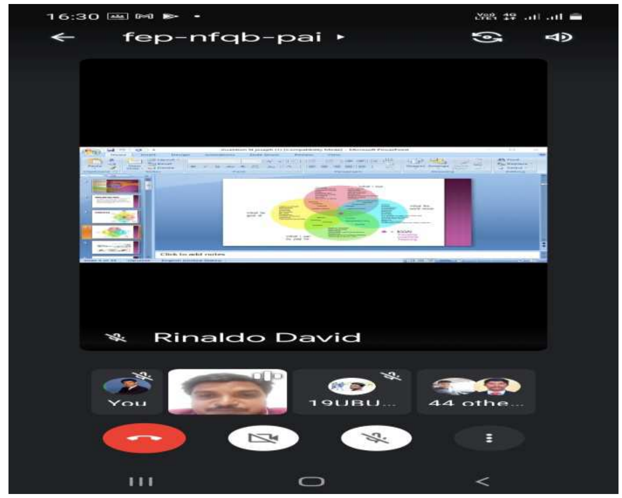
42 others
Rinaldo David



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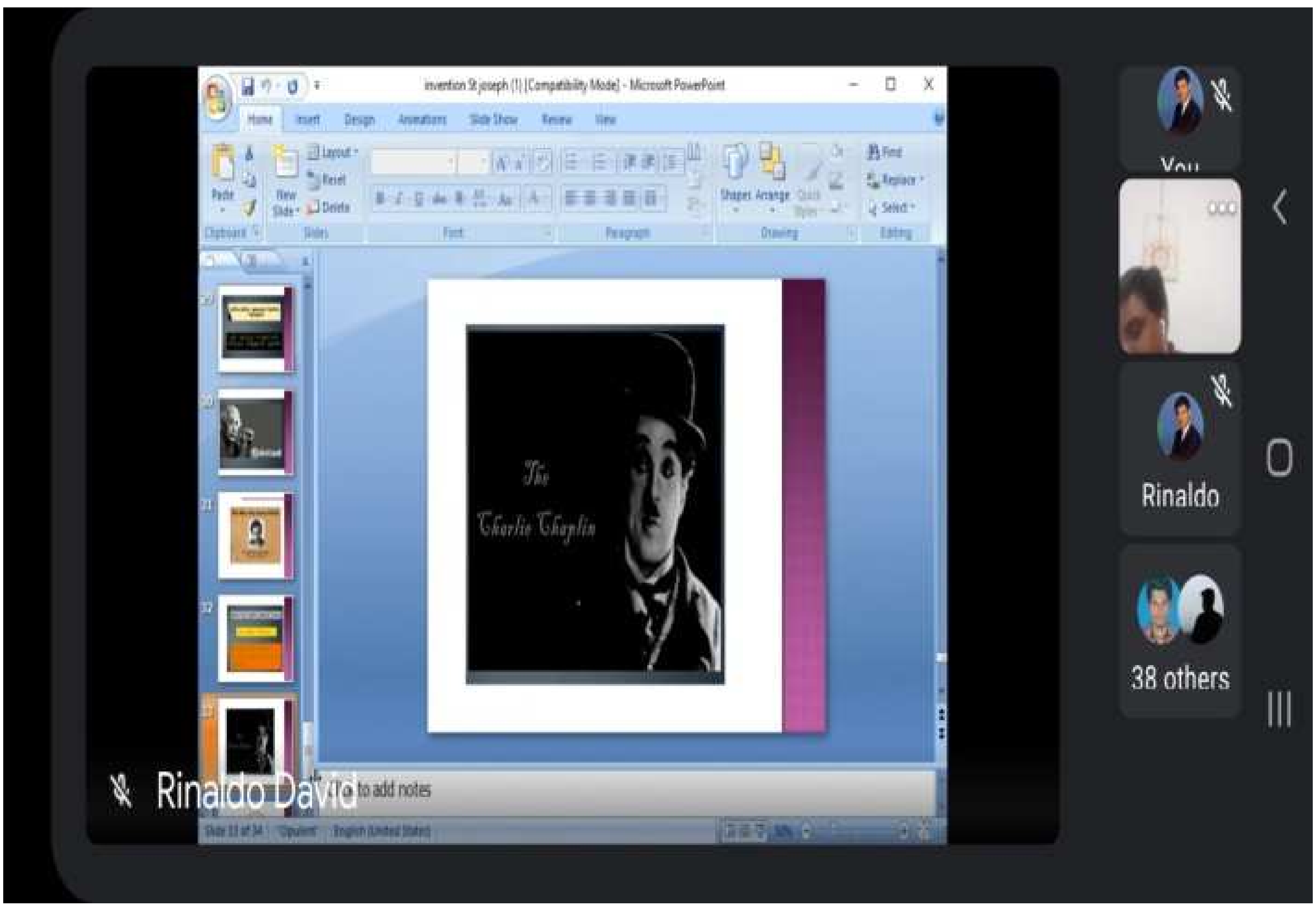
fep-nfqb-pai
Rinaldo David
You
19UBU...
44 othe...



invention St joseph (1) [Compatibility Mode] - Microsoft PowerPoint
Home
Insert
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Animations
Slide Show
Review
View
The
Charlie Chaplin
You
Rinaldo
38 others
Rinaldo David

Timestamp	Email Address	Score	Name	Contact Number	District	STATE	Are You a ---------?	Department	Year of Study	Programme Contents Me	Programme was useful fo	Facilitator's Explanation o	Any Other Suggestions
3/25/2021 16:58:44	shanmugamrvsims@gmail.com		Shanmugam	9791990175			Faculty		Not Applicable	5	Yes	5	Nil
3/30/2021 17:12:12	antonymalphilo95@gmail.com		Antony Malphilo	06379154407	Kanyakumari	Tamilnadu	UG Student	BBA	III Year	4	Yes	5	No
3/30/2021 17:12:43	santhoshsandy1220@gmail.com		SANTHOSH SIVAN N	9786306431	TRICHY	TAMILNADU	UG Student	BBA	III Year	4	Yes	5	Nothing
3/30/2021 17:12:46	josephpradeep8@gmail.com		Arockia Pradeep J	07598341084	Trichy	Tamilnadu	UG Student	BBA	III Year	5	Yes	5	Good
3/30/2021 17:12:48	18ubu635a@gmail.sjctni.edu		Manuel Fernando	09790483421	Tutucorin	Tamil Nadu	UG Student	BBA	III Year	5	Yes	5	Very useful
3/30/2021 17:12:59	19ubu601@mail.sjctni.edu		Sethu Rama Lingam T	09360059396	Tenkasi	Tamilnadu	UG Student	Bba	II year	4	Yes	4	No
3/30/2021 17:13:04	nanthu12072000@gmail.com		S.NANTHAKUMAR	+916379393371	Ariyalur	Tamilnadu	UG Student	BBA	II year	5	Yes	5	No
3/30/2021 17:13:06	19ubu606@mail.sjctni.edu		R Santhoshkumar	09344463131	Tiruchirappalli	Other	UG Student	BBA	II year	4	Yes	4	No
3/30/2021 17:13:07	19UBU619@mail.sjctni.edu		Martin Joseph. M	8248778312	Salem	Tamil Nadu	UG Student	Bba	II year	4	Yes	3	No
3/30/2021 17:13:20	18ubu657a@mail.sjctni.edu		Allwin Immanuvel M	9489705112	Karaikal	Pondicherry	UG Student	Business Administration (I	III Year	4	Yes	4	Nothing
3/30/2021 17:13:26	varathajohn555@gmail.com		Varatharajan C	6374478101	TIRUCHIRAPPALLI	Tamil Nadu	UG Student	BBA	III Year	5	Yes	5	Good
3/30/2021 17:13:27	18ubu538a@mail.sjctni.edu		Suriya J	7373514151	Trichy	Tamilnadu	UG Student	Business administration	III Year	4	Yes	4	No
3/30/2021 17:13:27	19ubu527@mail.sjctni.edu		Mohammed Sulaiman.A	6382738832	Dindigul	Tamilnadu	UG Student	Businesses administratior	II year	4	Yes	4	Good
3/30/2021 17:13:33	19ubu601@mail.sjctni.edu		Sethu Rama Lingam T	09360059396	Tenkasi	Tamilnadu	UG Student	Bba	II year	3	Yes	3	No
3/30/2021 17:13:33	stshariharan05@gmail.com		Hari Haran	09865743109	Trichy	Other	UG Student	BBA	II year	5	Yes	5	No
3/30/2021 17:13:46	aljinvj66@gmail.com		Aljin.V.J	8270204508	Kanyakumari	Tamilnadu	UG Student	BBA	III Year	4	Yes	4	No
3/30/2021 17:13:50	rajamohan25500@gmail.com		Rajmohan Mohan	09080452262	Trichy	Tamil nadu	UG Student	BBA	III Year	5	Yes	4	No
3/30/2021 17:13:55	19ubu519@mail.sjctni.edu		Bala Murugan	6369811327	Sivagangai	TAMILNADU	UG Student	Business Administration	II year	4	Yes	4	Clear explanation
3/30/2021 17:13:56	19ubu547@mail.sjctni.edu		MOHAMED ASLAM A E	6380228530	thiruvarur	Tamilnadu	UG Student	buisness administration	II year	5	Yes	4	no
3/30/2021 17:14:02	19ubu505@gmail.sjctni.edu		Rex Rex	07639157603	Kallakurichi	Other	UG Student	Business Administration	II year	4	Yes	4	No
3/30/2021 17:14:10	19ubu656@mail.sjctni.edu		Dhayanithi G	9123528234	Trichy	India	UG Student	B.B.A	II year	5	Yes	5	Nice
3/30/2021 17:14:11	19ubu635@mail.sjctni.edu		Ezhilamudhan	07092094355	Thanjavur	Tamil Nadu	UG Student	Department of Business A	II year	4	Yes	4	No
3/30/2021 17:14:24	rameshmukesh146@gmail.com		Mukesh N	8111084814	Trichy	Tamilnadu	UG Student	Business administration	III Year	5	Yes	5	No
3/30/2021 17:14:31	18ubu608a@sjctni.mail		Surya Kris K	9600820574	Trichy	Tamil Nadu	UG Student	BBA	III Year	1	No	2	Need eminent speakers
3/30/2021 17:14:35	19ubu568@mail.sjctn.edu		Akash S	9543094325	Kanyakumari	Tamil Nadu	UG Student	BBA	II year	3	Yes	4	No, i don't have any suggestions.
3/30/2021 17:14:37	19ubu624@mail.sjctni.edu		T logeshwar	7373851852	Trichy	Tamil Nadu	UG Student	Bba	II year	4	Yes	4	No, good experience
3/30/2021 17:14:47	neymarroshan13@gmail.com		I.Roshan	6385475644	Trichy	Tamilnadu	UG Student	BBA	II year	4	Yes	5	Yes
3/30/2021 17:14:50	19ubu521@mail.sjctni.edu		Theju Daniel. J	9150153220	Dindigul	Tamilnadu	UG Student	BBA	II year	4	Yes	5	No suggestions
3/30/2021 17:15:05	19UBU545@mail.sjctni.edu		JASPER INFANT A	9095155118	Trichy	Tamilnadu	UG Student	BBA	II year	5	Yes	5	Totally it is useful.
3/30/2021 17:15:33	michealdalwin1602@gmail.com		Micheal Dalwin.A	8610461051	sivaganga	TamilNadu	UG Student	BBA	II year	4	Yes	4	.
3/30/2021 17:15:44	18ubu611a@mail.sjctni.edu		SUESON. I	8903136778	Kanyakumari	Tamil Nadu	UG Student	BBA	III Year	4	Yes	4	I learn about something
3/30/2021 17:15:48	19ubu566@mail.sjctni.com		Viginesh. S	6383616854	Trichy	Tamilnadu	UG Student	BBA	II year	5	Yes	4	Nothing
3/30/2021 17:15:56	krishnadoss0411@gmail.com		Krishnadoss	8056370634	Thanjavur	Tamilnadu	UG Student	BBA	III Year	4	Yes	4	Good
3/30/2021 17:16:50	19ubu512@gmail.com		Francis Mamro	09360659576	Kanyakumari	Tamilnadu	UG Student	BBA	II year	4	Yes	5	No
3/30/2021 17:17:00	velan18072001@gmail.com		Velan	6379582406	Trichy	Tamilnadu	UG Student	BBA	III Year	3	Yes	3	No
3/30/2021 17:17:17	19ubu564@mail.sjctni.edu		Joe Francis S	8056660019	Trichy	Tamil nadu	UG Student	BBA	II year	5	Yes	5	Very useful
3/30/2021 17:19:05	jochristober3004@gmail.com		Jochristober	6385713296	Ramanathapuram	Tamil Nadu	UG Student	BBA	II year	3	Yes	3	No
3/30/2021 17:21:01	paulvineeth01@gmail.com		Ashwin Vineeth	8523985347	Cuddalore	Tamilnadu	UG Student	BBA	II year	5	Yes	5	.
3/30/2021 17:21:14	18ubu612a@mail.sjctni.edu		MANIKANDAN S	06385808987	Karur	Other	UG Student	BBA	III Year	4	Yes	5	Useful
3/30/2021 17:21:45	19ubu565@mail.sjctni.edu		dharik anwar	9345831214	TRICHY	TAMILNADU	UG Student	bba	II year	2	Yes	3	NOTHING
3/30/2021 17:21:57	hariharasudaan007@gmail.com		A.HARI HARA SUDAAN	7904321903	Trichy	Tamil nadu	UG Student	Bba	II year	4	Yes	4	Nothing
3/30/2021 17:22:38	19ubu546@mail.sjctni.edu		SHREE JAYANTH SARA	09629774396	TENKASI	TAMILNADy	UG Student	BUSINESSES ADMINIST	II year	5	Yes	5	NICE AND USEFUL SESSION TO US.
3/30/2021 17:23:58	19ubu654@mail.sjctni.edu		S.Naveen	7639081097	Dindigul	Tamilnadu	UG Student	BBA	II year	4	Yes	4	Very good
3/30/2021 17:24:04	akstyles36@gmail.com		Arun Karthi	06369698061	Trichy	Tamil nadu	UG Student	BBA	II year	4	Yes	5	Keep it up
3/30/2021 17:25:48	nithishsurya.nk@gmail.com		Nithish Kumar R	9566929864	Thanjavur	Tamil Nadu	UG Student	BBA	III Year	4	Yes	4	It was overall a good learning experience
3/30/2021 17:26:46	kevinandrew665@gmail.com		Kevina andrew R	9791693695	Trichy	Tamilnadu	UG Student	BBA	III Year	4	Yes	4	Excellent
3/30/2021 17:26:58	18ubu605a@mail.sjctni.edu		B SRIHARISUGAN	9597971640	Namakkal	Tamil Nadu	UG Student	Bachelor of BUSINESS A	III Year	5	Yes	5	No
3/30/2021 17:27:01	bharathi19648@gmail.com		R. Bharathi	7867931537	Karur	Tamil Nadu	UG Student	BBA	II year	5	Yes	3	No
3/30/2021 17:28:21	karanmuthur2224@gmail.com		Karan s	9943114702	Sivagangai	Tamil nadu	UG Student	BBA	III Year	4	Yes	5	Nothing
3/30/2021 17:29:12	dineshbaburohit@gmail.com		Rohit	9488769168	Tiruchirappalli	Tamil Nadu	UG Student	Business Administration	III Year	4	Yes	4	No
3/30/2021 17:29:31	19ubu529@mail.sjctni.edu		GOWTHAM M	9976281136	Karur	Tamilnadu	UG Student	BBA	II year	3	Yes	3	no
3/30/2021 17:29:51	18ubu523a@mail.sjctni.edu		Abdul Faseet Khan B.K	06384637746	Trichy	Tamilnadu	UG Student	Business Administration	III Year	5	Yes	4	It's nice and we'll understandable
3/30/2021 17:30:56	18ubu549a@mail.sjctni.edu		jerson ebinesh	08524985894	Pudukkottai	tamil nadu	UG Student	BBA	III Year	5	Yes	5	it's very good
3/30/2021 17:31:13	19ubu525@gmali.Sjctni.edu		s monanpandy	7904738664	trichy	tamilnadu	UG Student	bba	II year	3	Yes	3	No
3/30/2021 17:31:47	18ubu567a@mail.sjctni.edu		Antony samy.I	7448548484	Trichy	Tamilnadu	UG Student	BBA	III Year	1	Yes	2	சிறப்பு
3/30/2021 17:32:42	18ubu572a@mail.sjctni.edu		Bharathiraja S	06379273290	Perambalur	Tamil Nadu	UG Student	BBA	III Year	4	Yes	4	Good
3/30/2021 17:32:53	arunkumarajalingam028@gmail.com		Arun Kumar	9789334377	Trichy	Tamilnadu	UG Student	BBA	III Year	5	Yes	5	No suggestions
3/30/2021 17:34:34	18ubu543a@mail.sjctni.edu		Ranjith Kumar J	07094563792	Thanjavur	TamilNadu	UG Student	BBA	III Year	1	Yes	1	No
3/30/2021 17:37:39	18ubu546a@mail.sjctni.edu		jone amala nathan.a	7867989559	trichy	tamil nadu	UG Student	BBA	III Year	5	Yes	5	very good
3/30/2021 17:39:21	18UBU562a@gmail.com		SANDHANA YASOR. J	08940328101	Thanjavur	Tamilnadu	UG Student	BBA	III Year	3	Yes	4	Good
3/30/2021 17:39:36	18ubu656a@mail.sjctni.edu		Stephen s	9843920238	Puthukkotti	Tamilnadu	UG Student	BBA	III Year	5	Yes	5	No
3/30/2021 17:40:19	19ubu676@mail.sjctni.edu		Harish	07397178301	ramanathapuram	Tamil Nadu	UG Student	BBA	II year	4	Yes	4	No
3/30/2021 17:45:25	akash.jose1603@gmail.com		Akash jose	6381740560	Cuddalore	Tamil Nadu	UG Student	BBA	III Year	5	Yes	4	It's good
3/30/2021 17:45:33	18ubu646a@mail.sjctni.edu		Lakshmanan	9677307781	Tiruchirapalli	Tamilnadu	UG Student	BBA	III Year	5	Yes	5	Really great
3/30/2021 17:46:39	19ubu541@mail.sjctni.edu		Pitchai pandi M	9952215842	Dindigul	Tamilnadu	UG Student	BBA	II year	4	Yes	2	It is a useful program
3/30/2021 17:55:47	19ubu538@mail.sjctni.edu		Venniz Joe Roman . J	09842090604	Trichy	Tamil Nadu	UG Student	Bachelor Of Business Adr	II year	5	Yes	5	.
3/30/2021 17:59:26	kgokul005@gmail.com		Gokul K	9629950485	Virudhunagar	Tamilnadu	UG Student	Business Administration	II year	4	Yes	4	Nothing
3/30/2021 17:59:42	18ubu542a@mail.sjctni.edu		SUDHA KANRAN	9952201155	Karur	Tamilnadu	UG Student	BBA	III Year	4	Yes	3	Good
3/30/2021 18:01:12	18ubu528a@mail.sjctni.edu		Alan Roshan	09498859528	Trichy	Other	UG Student	BBA	III Year	5	Yes	5	Gud
3/30/2021 18:02:08	19ubu523@mail.sjctni.edu		SRI KRISHNA S	08883287434	Trichy	Tamilnadu	UG Student	BBA	II year	5	Yes	4	It's fine and good.
3/30/2021 18:02:32	19ubu502@mail.sjctni.edu		Jacob Samuvel	09344400433	Trichy	Tamilnadu	UG Student	Bba	II year	4	Yes	4	Nothing
3/30/2021 18:15:22	19ubu561@mail.sjctni.edu		Titus Titus	08925429074	Dindigul	Other	PG Student	BBA	II year	3	Yes	4	No
3/30/2021 18:15:50	jerin1142@gmail.com		Antro jerin A	7094435251	Theni	Tamil Nadu	UG Student	BBA	III Year	3	Yes	4	No
3/30/2021 18:23:02	19ubu670@mail.sjctni.edu		Samuvel. P	6382457016	Trichy	Tamilnadu	UG Student	Bba	II year	3	Yes	1	Good ok
3/30/2021 18:31:14	headvindoss@gmail.com		Headvin doss j	9698344142	Trichy	Tamil nadu	UG Student	BBA	II year	4	Yes	4	Nothing
3/30/2021 18:34:58	vkmarunnithi@gmail.com		Arunnithi.M	9677886673	Trichy	Tamilnadu	UG Student	BBA 'A'	II year	4	Yes	5	Good
3/30/2021 18:56:57	19ubu635@mail.sjctni.edu		Ezhilamudhan	07092094355	Thanjavur	Tamil Nadu	UG Student	Department of Business A	II year	4	Yes	4	No
3/30/2021 19:00:30	randyarun8201@gmail.com		Antony Arun A	9786849567	Trichy	Tamil Nadu	UG Student	BBA	III Year	4	Yes	5	Good explanation
3/30/2021 19:01:13	varunncc72@gmail.com		19ubu571 Vairaperumal	07358927439	Trichy	Other	UG Student	BBA	II year	4	Yes	3	No
3/30/2021 19:07:29	19ubu532@mail.sjctni.edu		K Guru prasath	8248214474	Trichy	Tamil Nadu	UG Student	BBA	II year	5	Yes	5	No issuses
3/30/2021 19:19:44	19ubu623@mail.sjctni.com		Manoj Kumaaran	08110910016	Sivagangai	Other	UG Student	Bbw-b	II year	3	Yes	3	.
3/30/2021 19:20:38	veeramanivadivel23@gmail.com		Veeramani. v	9585967288	karur621313	tamilnadu	UG Student	BBAA	III Year	3	No	4	Human
3/30/2021 19:43:23	asathjanmohamed09321@gmail.com		Asath Asath	07904323288	ARIYALUR	TAMILNADU	UG Student	BBA	III Year	3	Yes	3	No
3/30/2021 19:49:35	18ubu516a@mail.sjctni.edu		REVANTH J	9566732398	Ramanathapuram	Tamil Nadu	UG Student	BBA	III Year	5	Yes	5	No
3/30/2021 20:11:24	19ubu610@mail.sjctni.edu		19UBU610_Ruban Joyvir	07708262448	Pudukkottai	Other	UG Student	BBA	II year	4	Yes	4	No
3/30/2021 21:59:44	18ubu553a@mail.sjctni.edu		PRIYAN	9600371230	Sivaganga	Tamil Nadu	UG Student	BBA	III Year	3	Yes	3	Good
3/30/2021 22:50:50	18ubu553a@mail.sjctni.edu		PRIYAN	9600371230	Sivaganga	Tamil Nadu	UG Student	BBA	III Year	3	Yes	3	Good
3/31/2021 11:40:13	18ubu572a@mail.sjctni.edu		Bharathiraja S	06379273290	Perambalur	Tamil Nadu	UG Student	BBA	III Year	4	Yes	4	Good